Scaling Phenomena

f.hasselman@pwo.ru.nl
a.bosman@pwo.ru.nl

https://www.ru.nl/bsi/research/group-pages/complex-systems-group/

Behavioural Science Institute
Pedagogical & Educational Sciences
Radboud University Nijmegen

Scaling phenomena

How long is the coast of Great-Britain?

Scaling phenomena

scale
11.5 x 200 = 2300 km

scale
28 x 100 = 2800 km

scale
70 x 50 = 3500 km

Length systematically depends on the size of the measurement stick you use!

https://en.wikipedia.org/wiki/Fractal_dimension

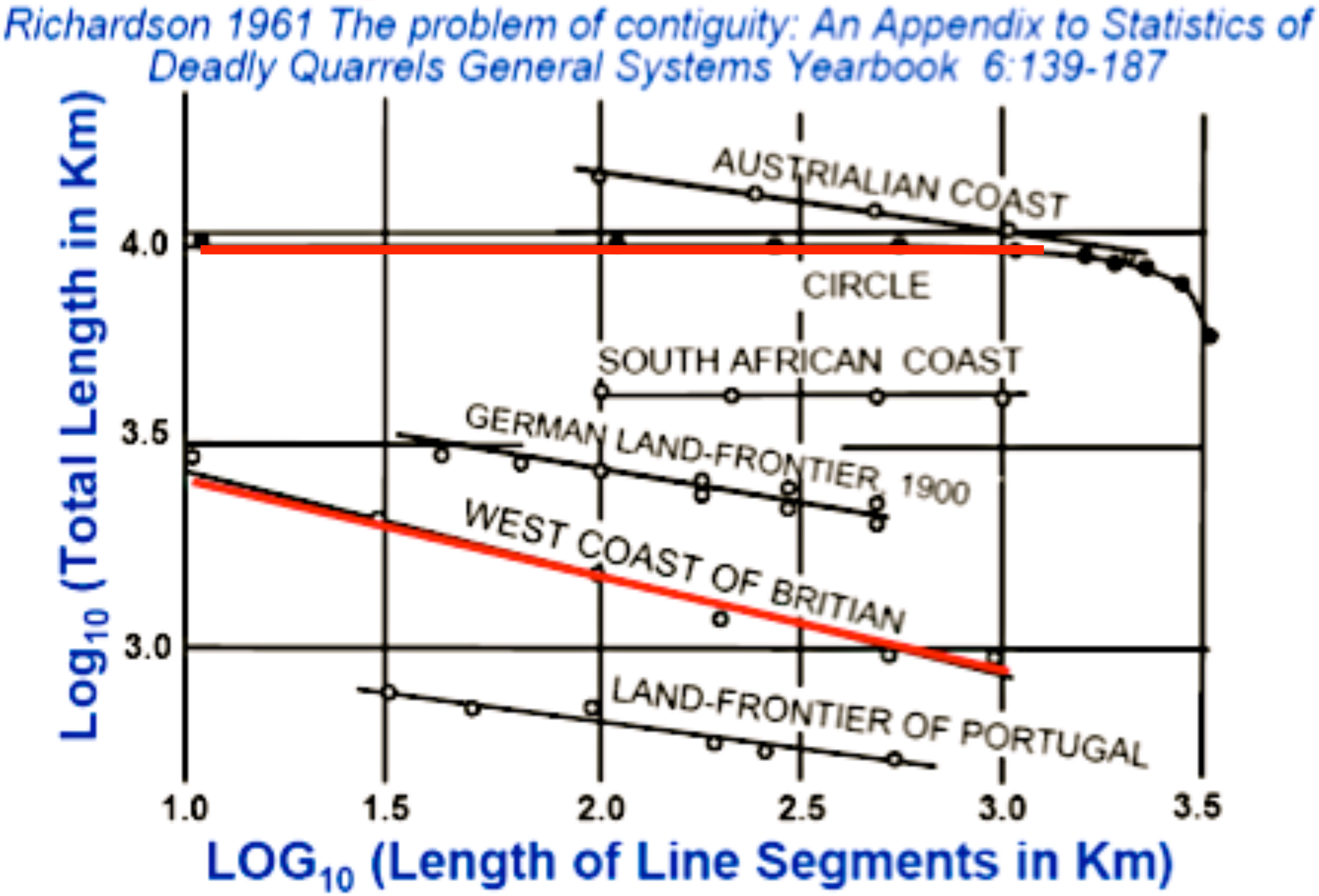
Scaling phenomena

"scaling of bulk with size"
(Theiler, 1990)

The formal answer to the question is:
"There is no characteristic scale at which the length of the coast of GB can be expressed"

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science, 156*(3775), 636–8.

How Long is the Coastline of Britain?



Scale invariance…

A *power law* scaling relation (**LOG scale**):
There is no characteristic length, just an indication of complexity

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science, 156*(3775), 636–8.

Scaling phenomena

Scaling relations can emerge with all kinds of observables
They inform about properties of the process / system under scrutiny

Earthquakes (Richter-Law)

frequency of occurrence ~ magnitude

Distribution of mass in the Universe

resolution ~ density

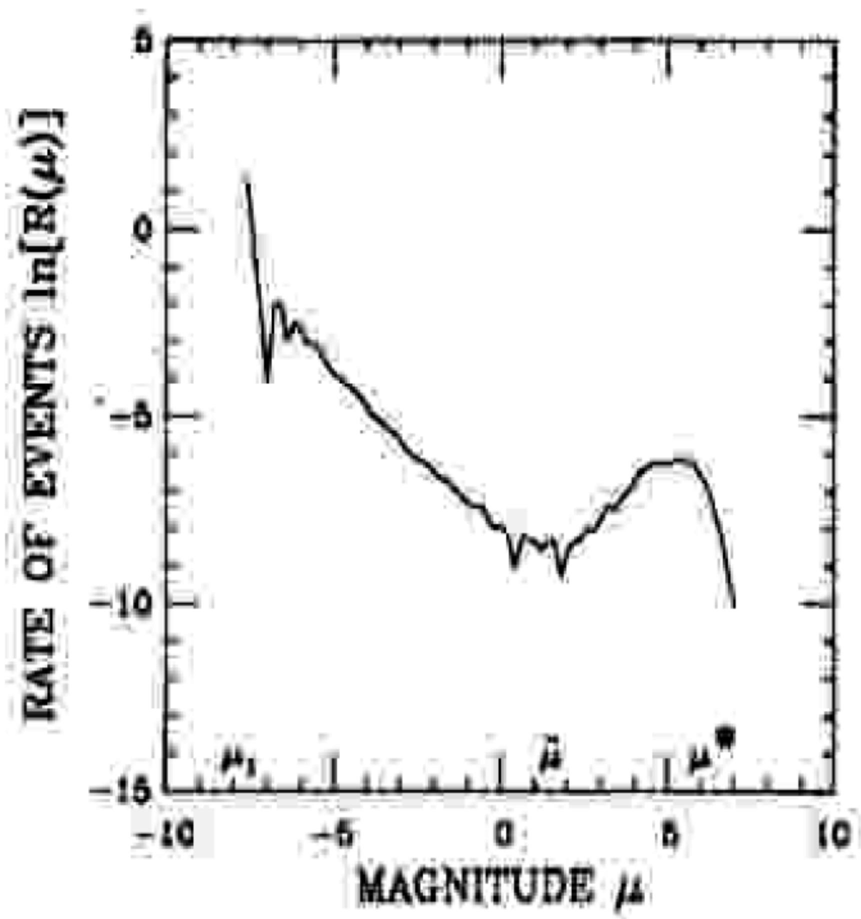
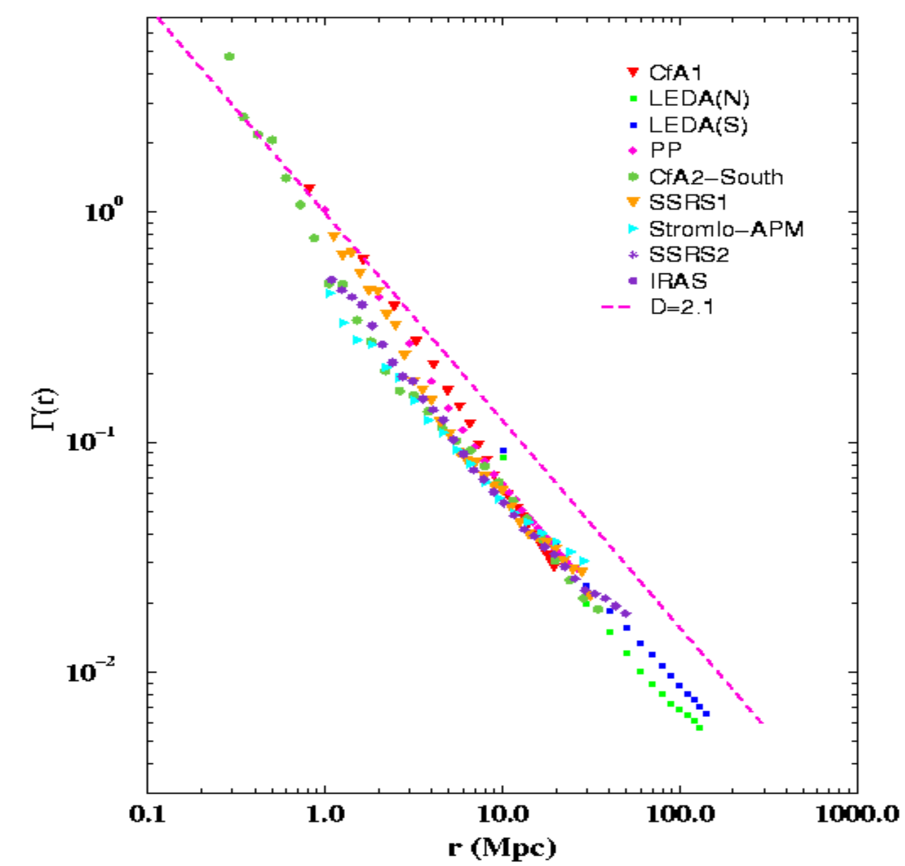


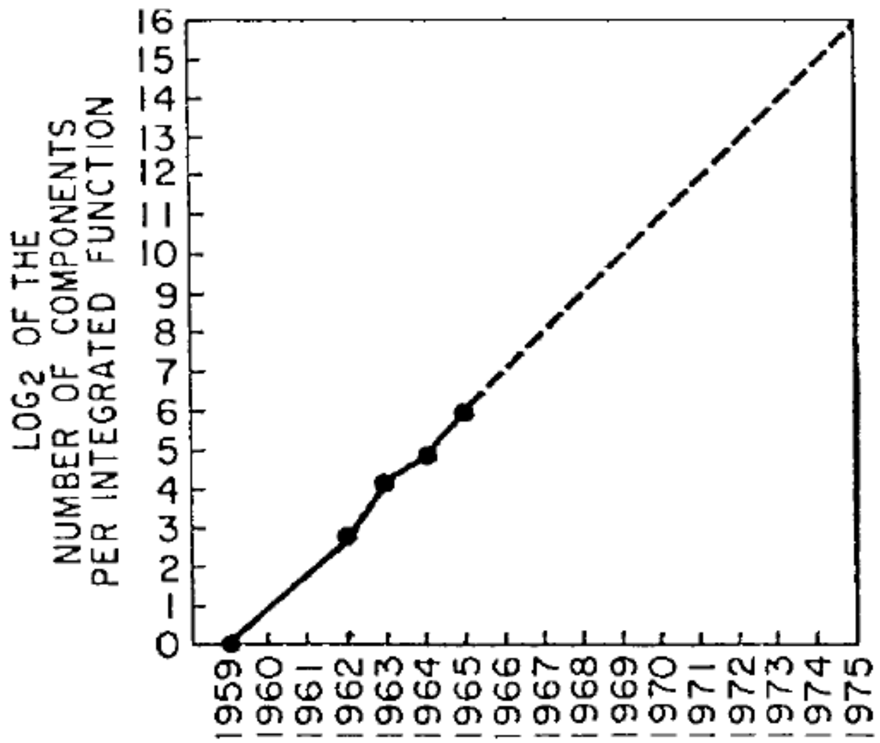
Figure 13: frequency distribution of the slip events (earthquakes) of magnitude μ taken from [53]. Notice the large bump that corresponds to an excess of events of high magnitude.



Scaling & Growth

Moore's Law:

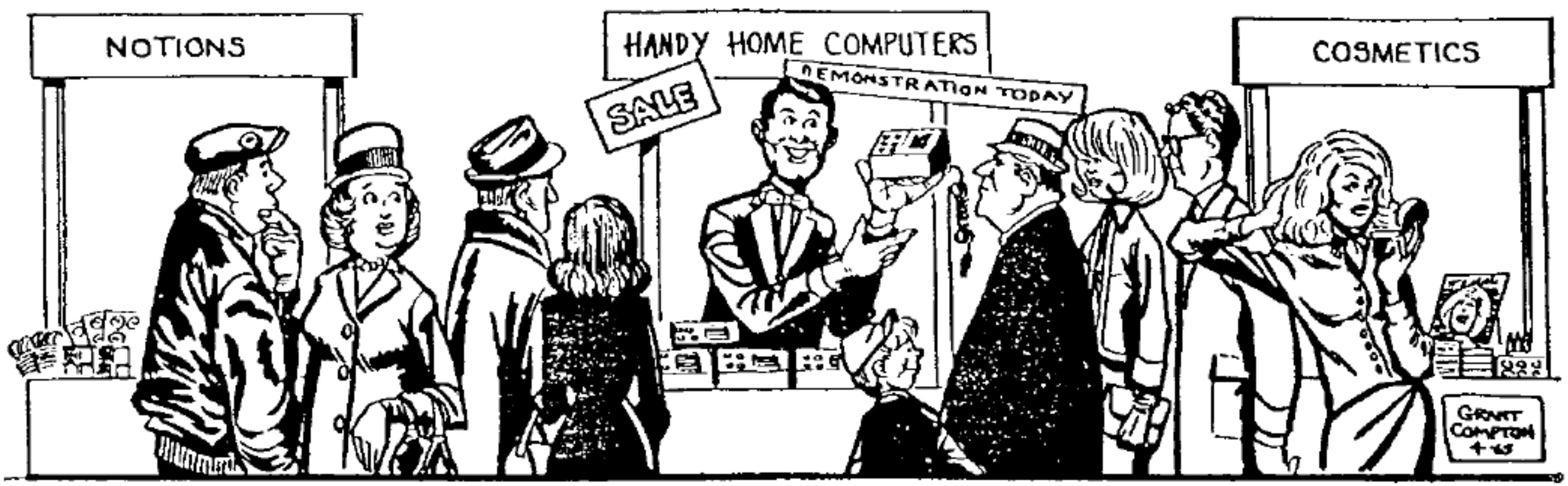
Predicted if speed of innovations in "cramming more components onto integrated circuits" kept up …



Moore's Law:

… we would soon be buying computers at the local market …

which apparently was a preposterous idea



Moore, Gordon E. (1965). "Cramming more components onto integrated circuits"

Behavioural Science Institute
Radboud University Nijmegen

VANAF WOENSDAG 26-02
ONZE AANBIEDINGEN
Tablet PC cover
6.99*
GSM AT-B26D
FM-RADIO
BLUETOOTH 2.1
OP=OP
49.99*
10" Tablet MD98516
16 GB GEHEUGEN
179.00*
ALDI TALK
Wireless speaker adapter
14.99*
Bevestigingsset
2.99*
Hortensia
4.79*
ALDI
Aperitiefbiscuits
0.99*
Basis voor soep
1.29*
Munt- of honingdrop
0.99*
Spijsbroodjes of appelflappen
1.19*
Mini-stroopwafels
1.89*
Smulgerechten
Chips patatje kapsalon of hete kip
0.99*
6-7/16
Index
Trefwoord

Kruidvat
voordeel magazine
Goed gekleed
DAMESJASJE M t/m XL
39.99 19.99
Stralende lippen
MAX FACTOR LIPSTICK, -GLOSS OF -LINER
ALLES VOOR € 10.00 PER STUK
Perfect haar
NIVEA HAARVERZORGING OF STYLING
1+1 GRATIS
Lekker schoon
ROBOTSTOFZUIGER
Dirt Devil
129.99 79.99
STEEDS VERRASSEND, ALTIJD VOORDELIG!
Geldig van dinsdag 25 februari t/m zondag 9 maart 2014

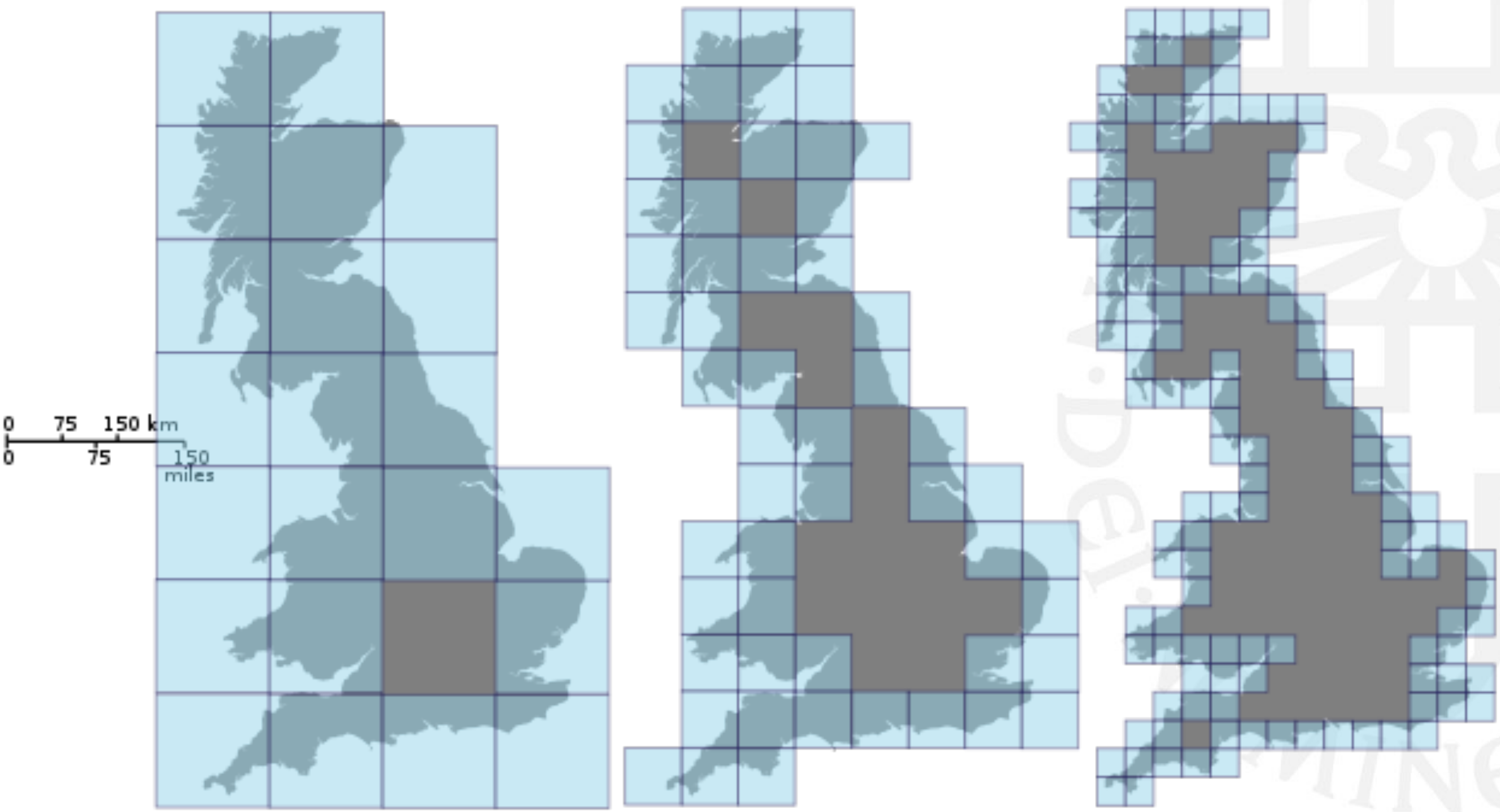
What is scaling?
Self-similarity & Self-affinity

Object looks roughly the same on all scales = (Statistical) **self-similarity** (“zoom similarity”)
(Statistical) self-similarity is observed after affine transformation = **self-affinity** (“warp similarity”)

Degree of invariance across scales = Dependencies/regularities/correlations across scales

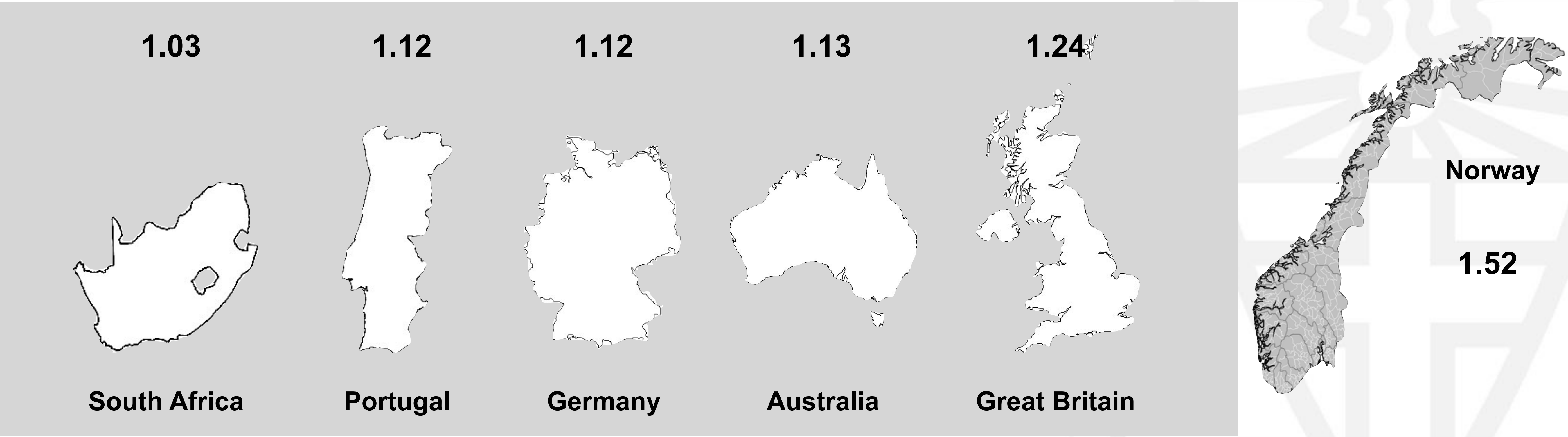
aka: “Nested scales”

How to describe scaling relations: Calculate a “fractional” dimension, e.g. box-counting dimension

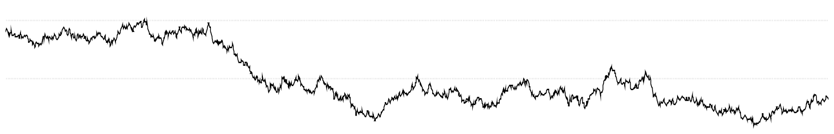


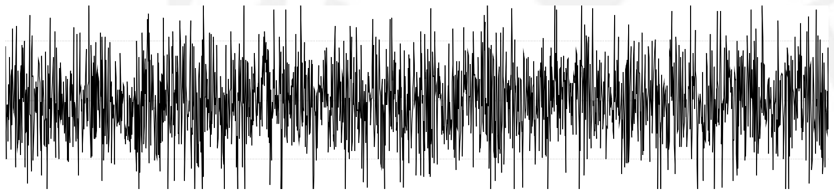
Scaling phenomena

Fractional dimension = “spill over” into next dimension
Associated to Processes & Properties



line-like → plane-like





Scaling phenomena

"Optimised" packing/filling

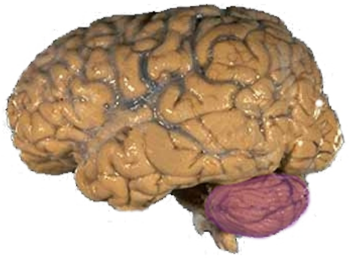
Packing Cubes or Spheres and Wrapping Blankets:

2D ~ 3D spatial scaling relations in nature:

Cauliflower fractal dimension = 2.33

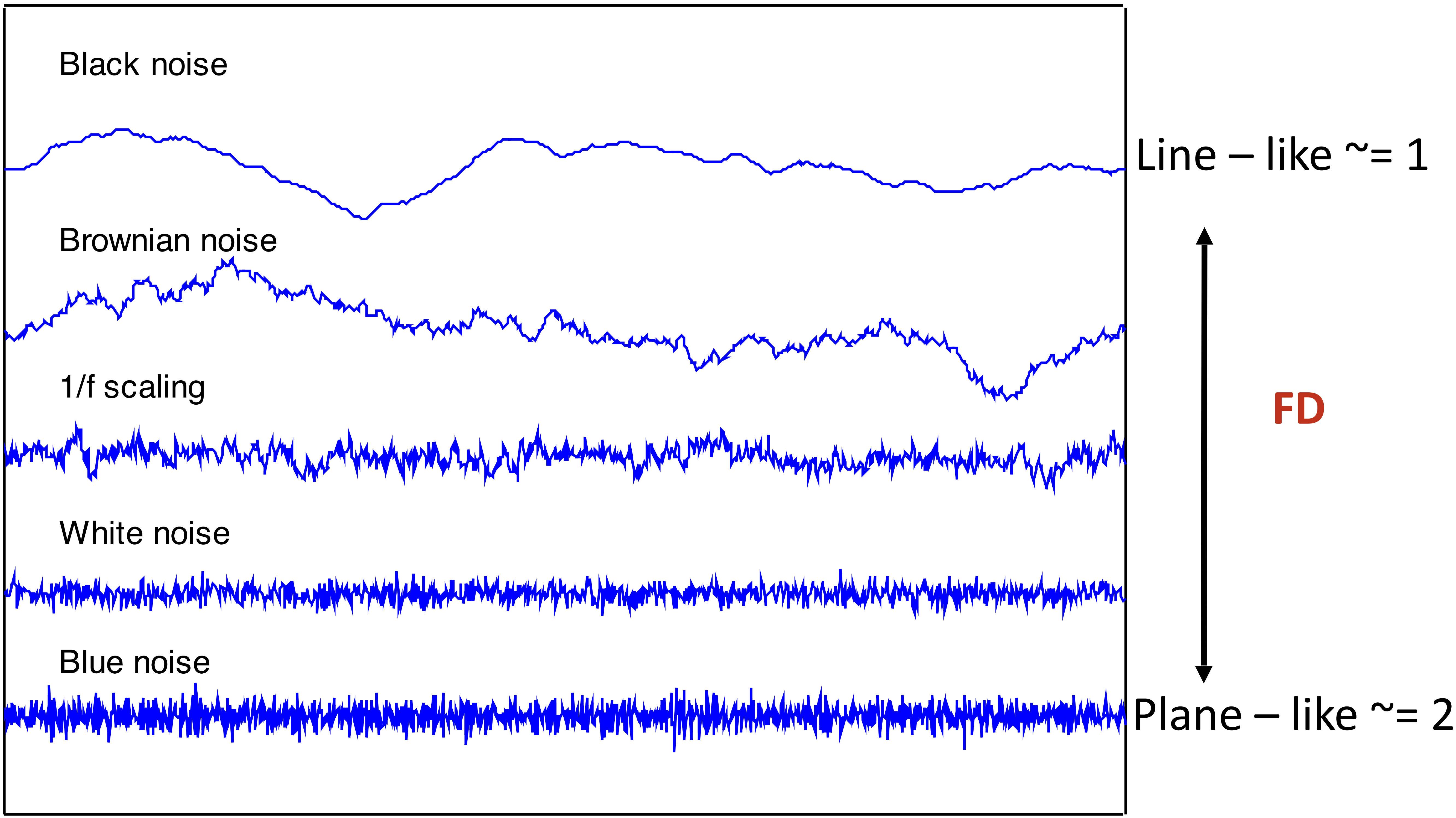
Surface of human lungs: 2.97

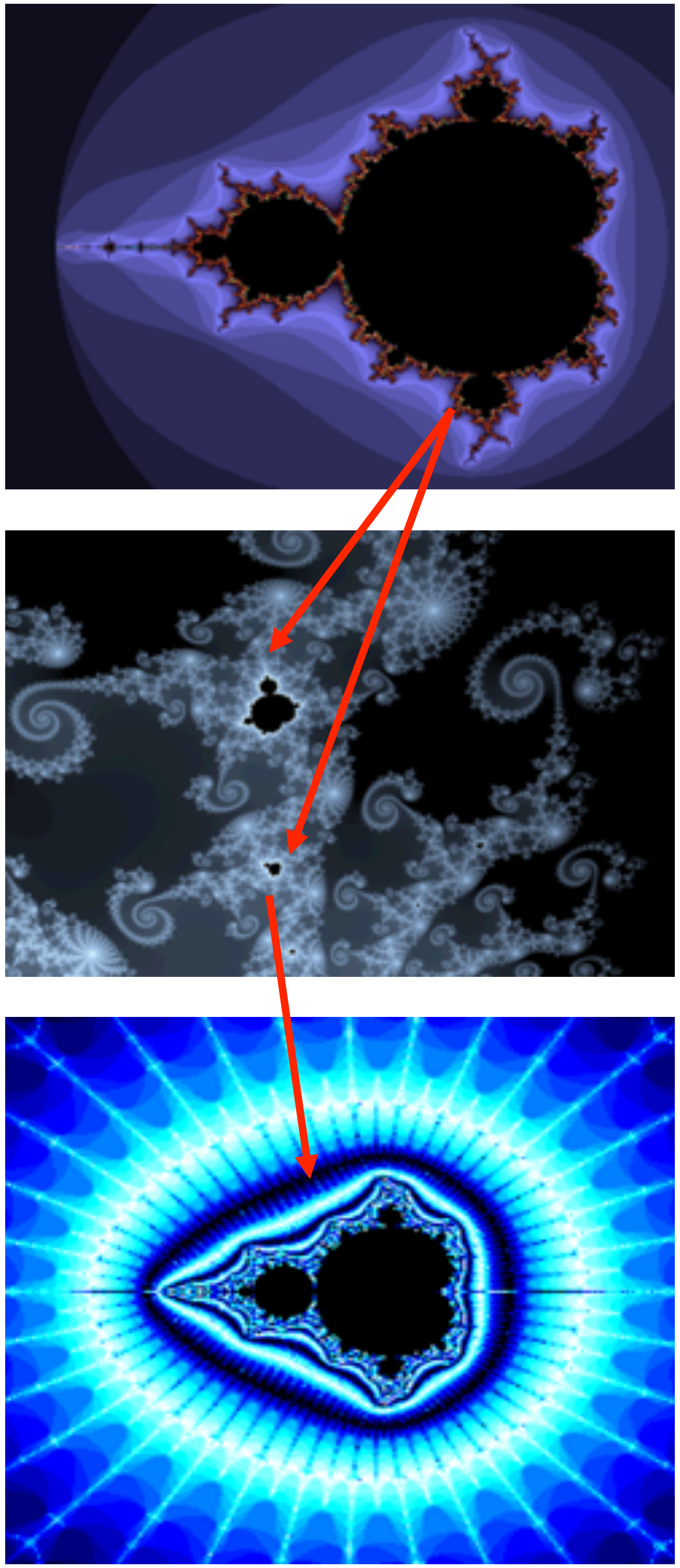
Surface of human brain: 2.79

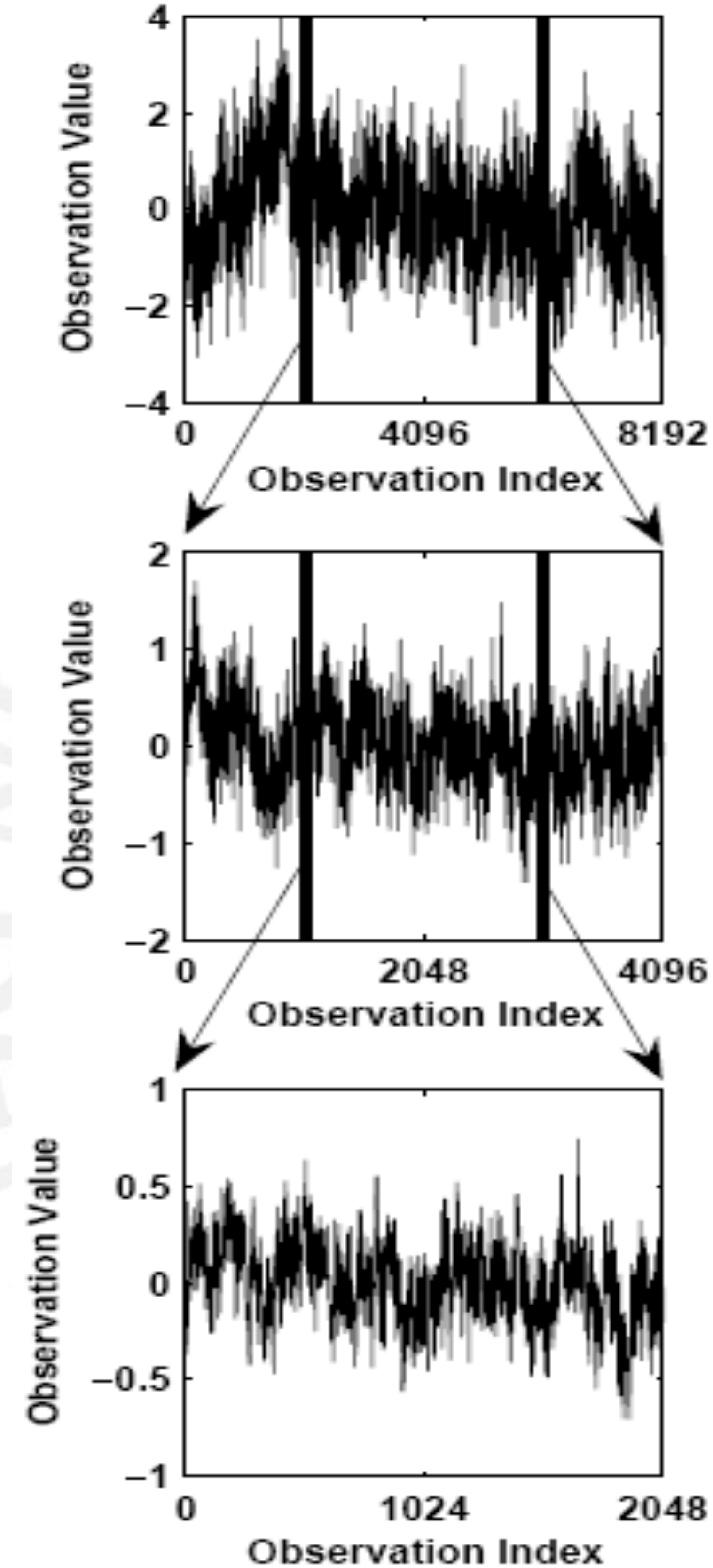


WHY OPTIMISED SURFACE AREA and not VOLUME?

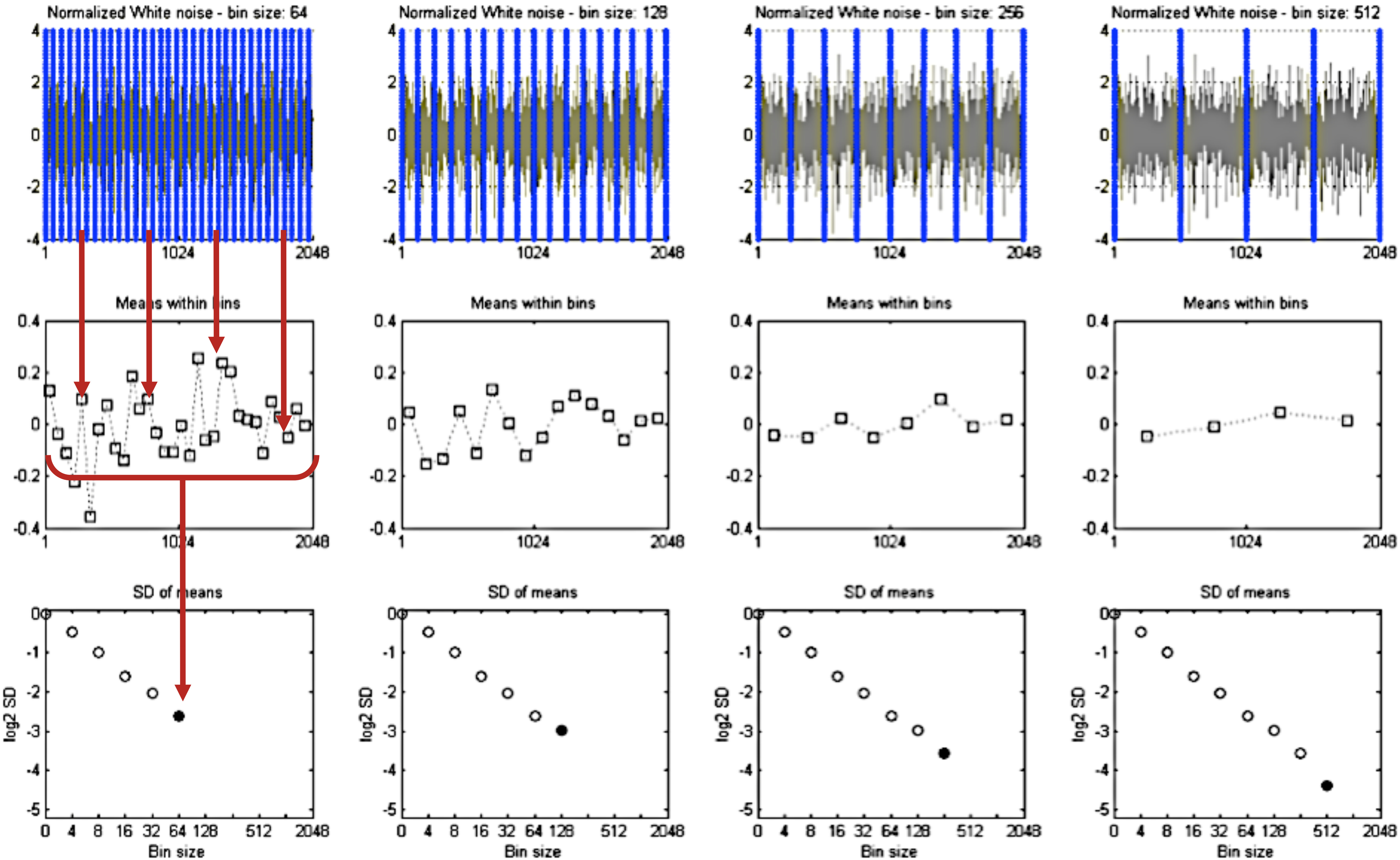
Scaling exponents reveal properties of data generating processes

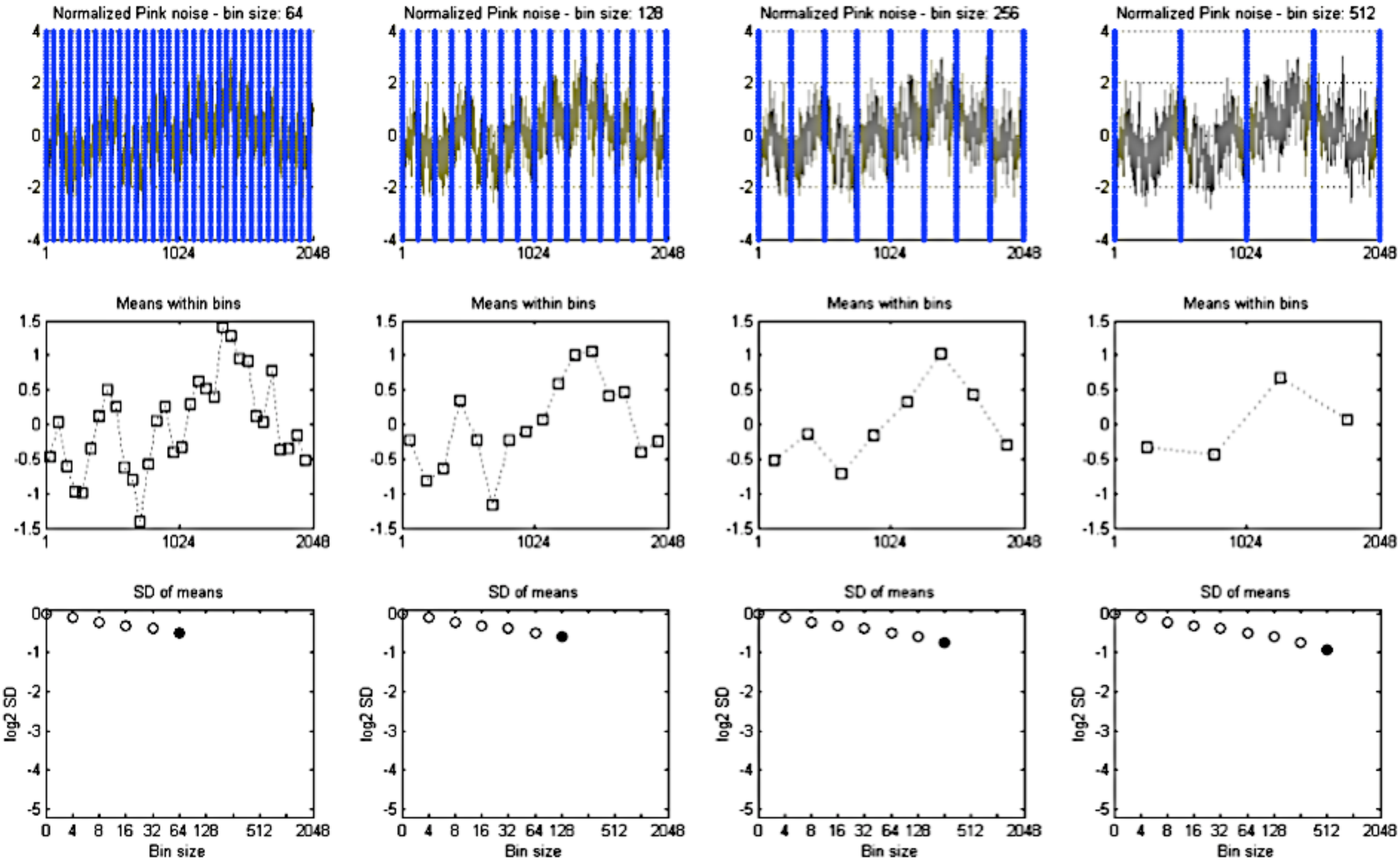


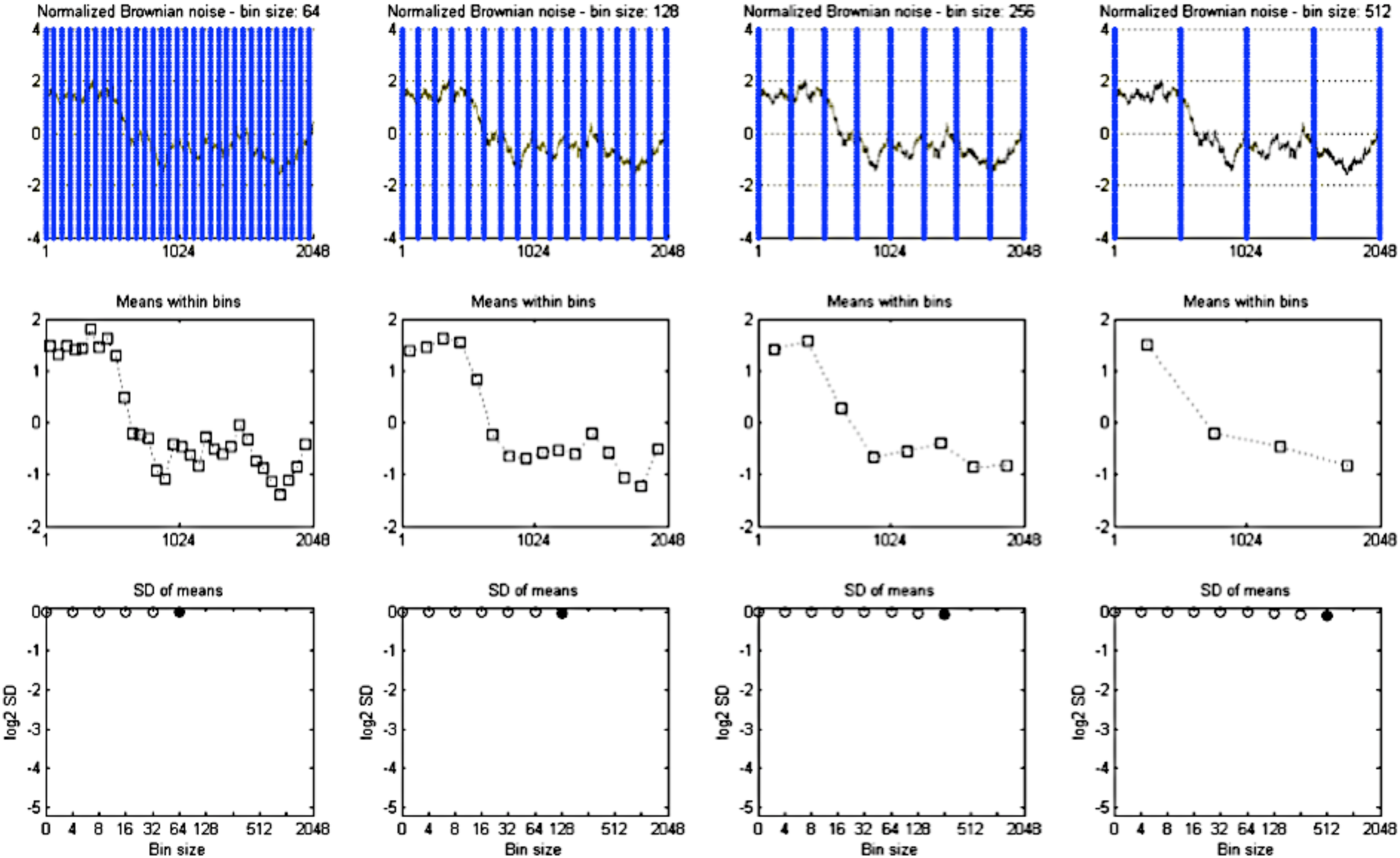




aka: “Fractal scaling”


Normalized White noise - bin size: 64
Normalized White noise - bin size: 128
Normalized White noise - bin size: 256
Normalized White noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size
0 4 8 16 32 64 128 512 2048
1 1024 2048


Normalized Pink noise - bin size: 64
Normalized Pink noise - bin size: 128
Normalized Pink noise - bin size: 256
Normalized Pink noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size


Normalized Brownian noise - bin size: 64
Normalized Brownian noise - bin size: 128
Normalized Brownian noise - bin size: 256
Normalized Brownian noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size
1
1024
2048
0
4
8
16
32
64
128
512

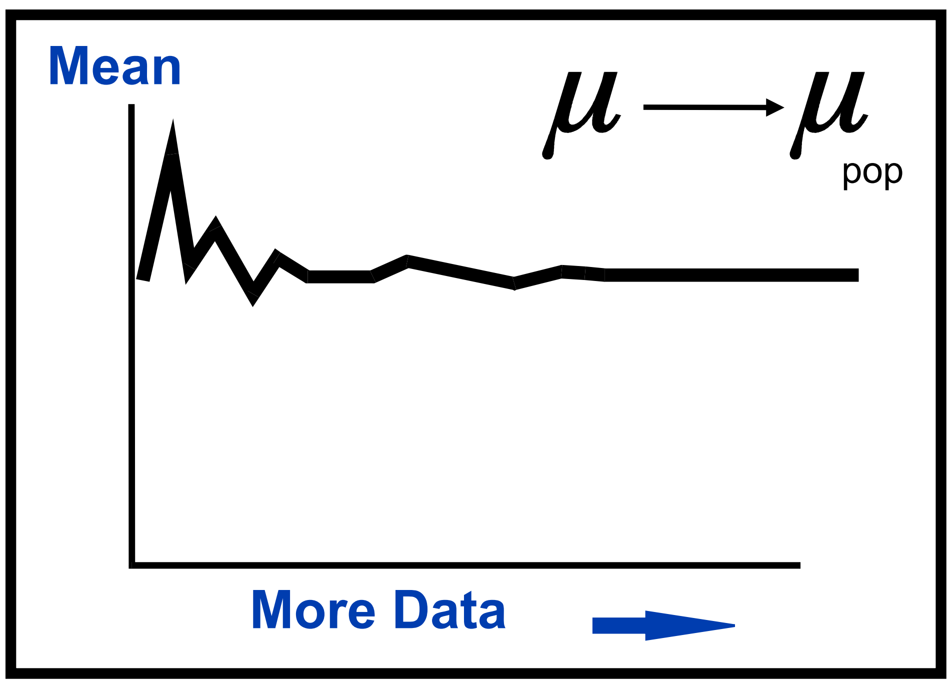
Scaling phenomena: Time scales

Independent observations of random variables

μ±σ are sufficient to characterise absence of dependencies in the data:

e.g. Expected value of **μ** for **N** = 100, given **σ**

N = ensemble size

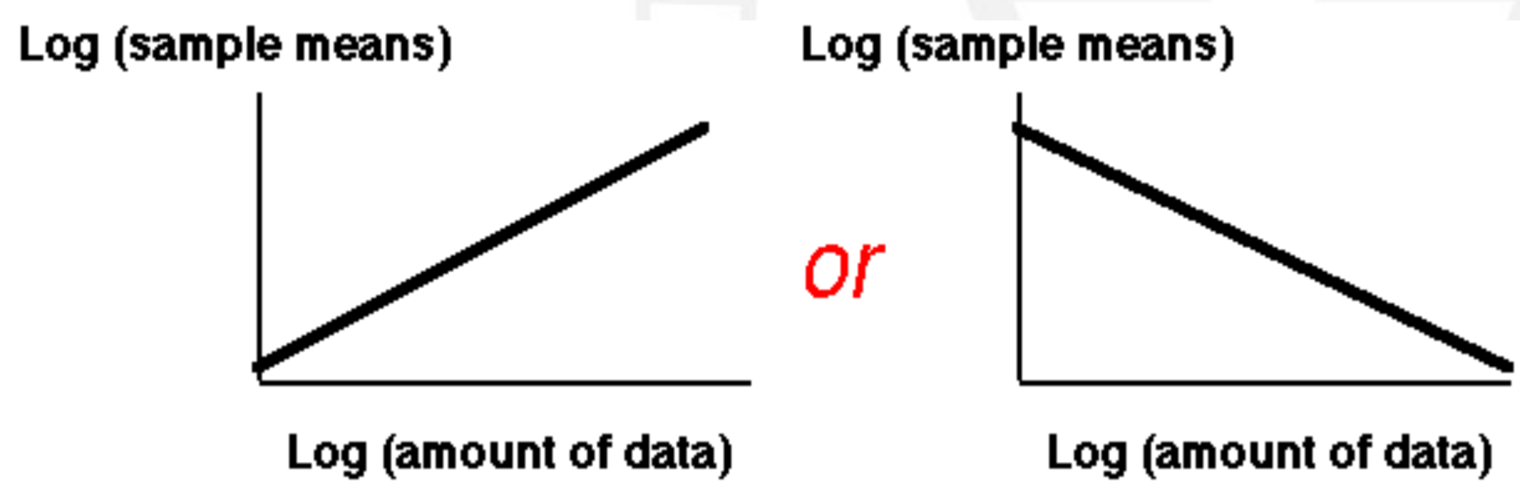


Interdependent observations across different scales

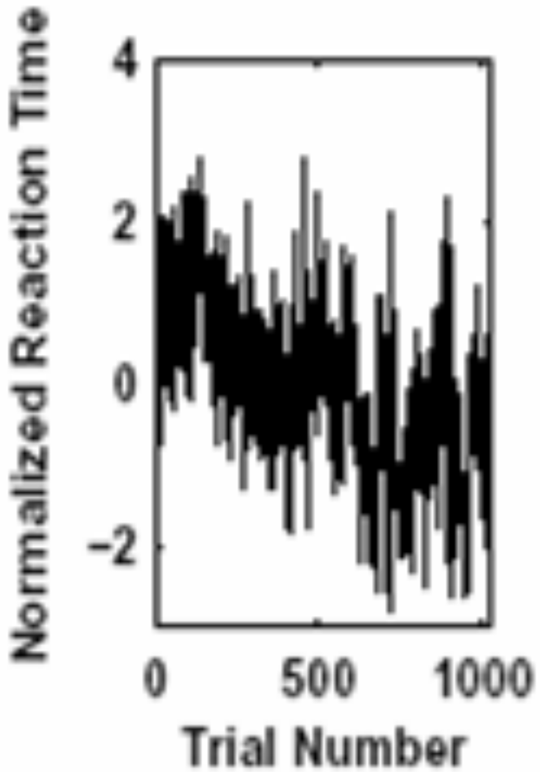
μ±σ are insufficient to characterise dependencies in the data:

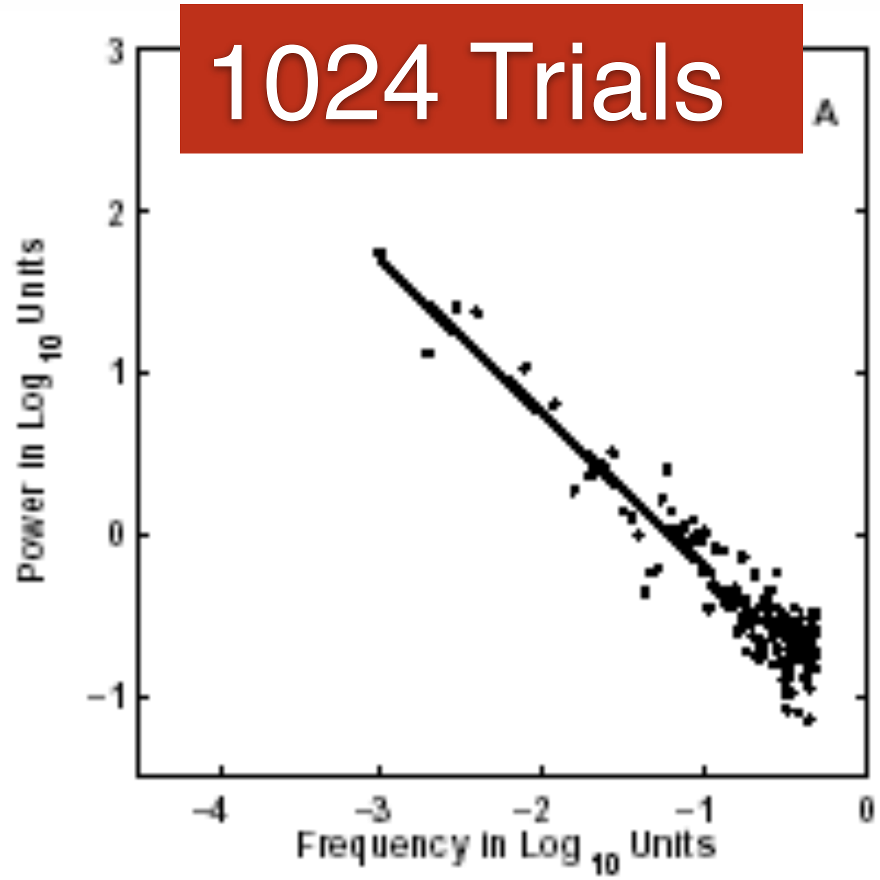
e.g. Sample estimates of **μ** change with **N**

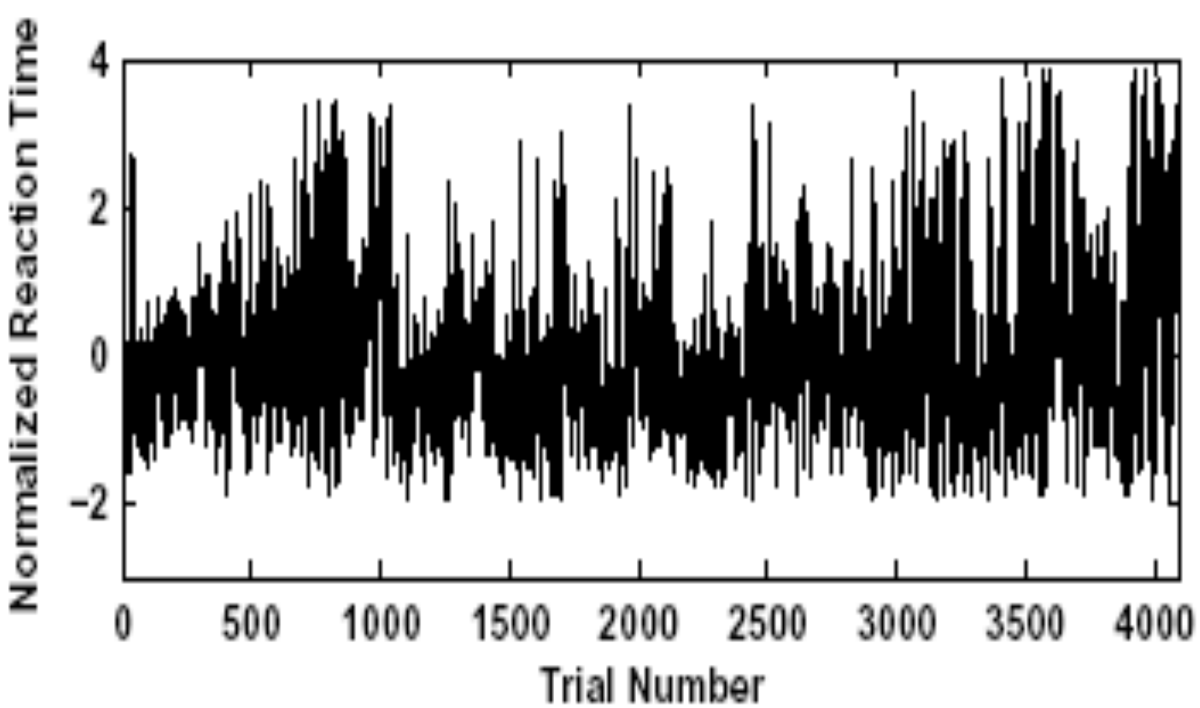
N = observation time

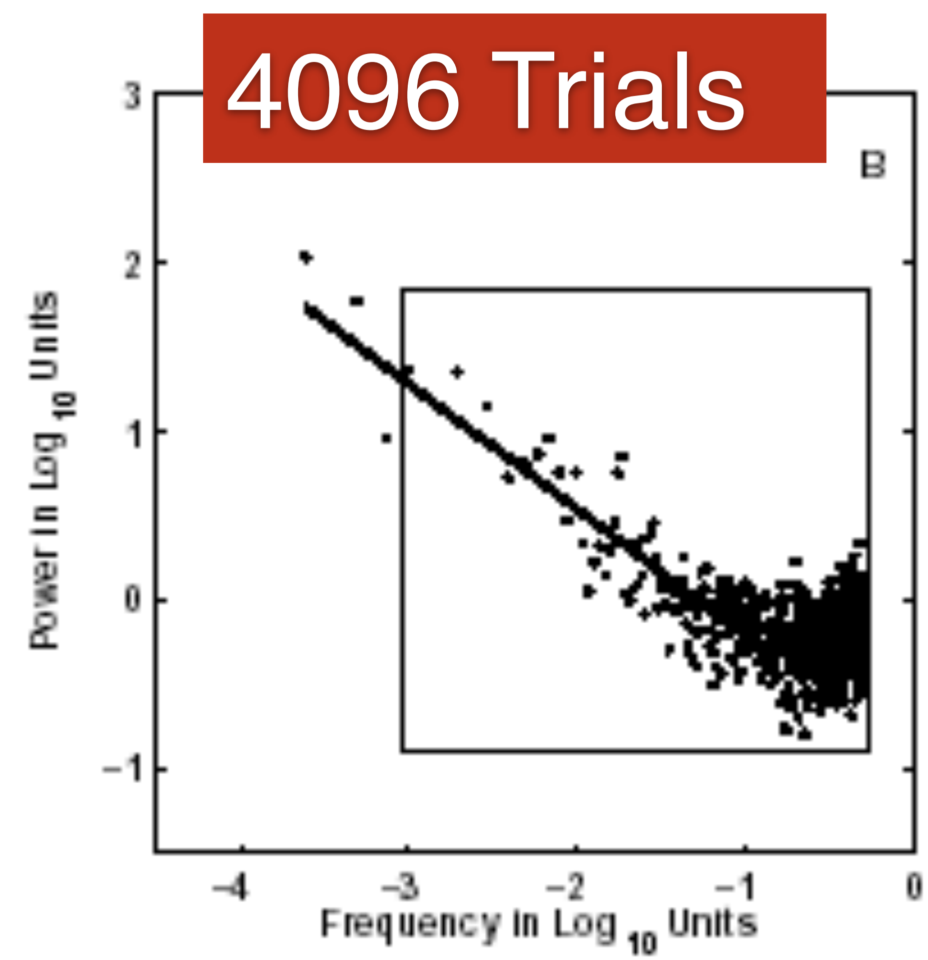


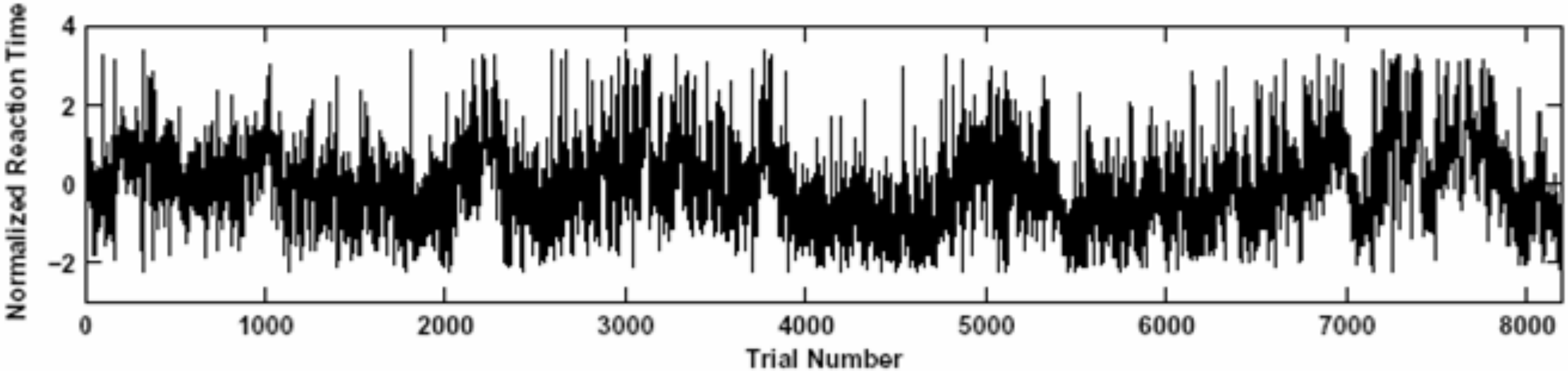
“Statistics”: More data = more variance



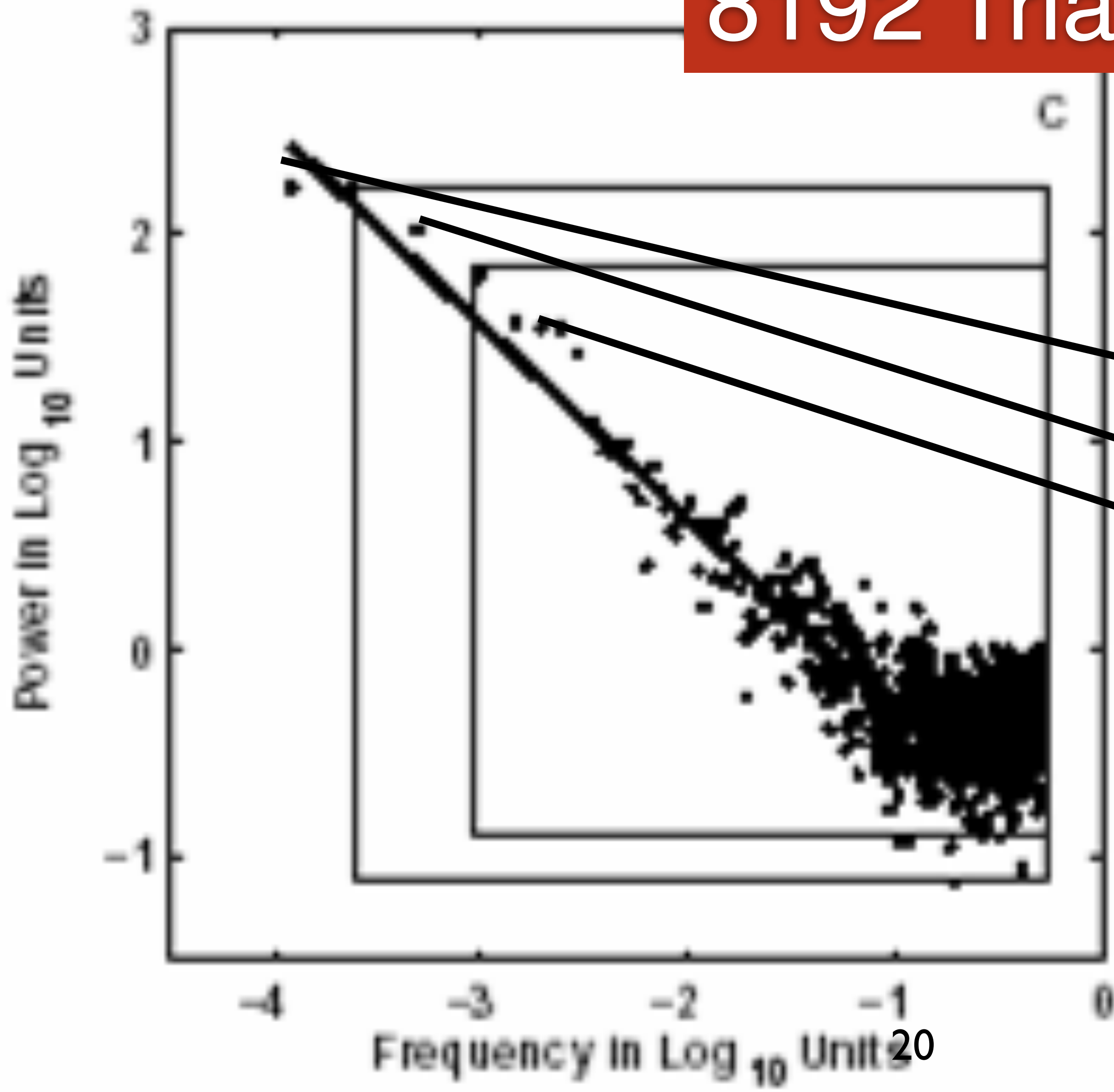








8192 Trials (3 HOURS)



More data points = more variability!

Scaling exponents reveal properties of data generating processes

Sixth International Conference
on Noise in Physical Systems

Proceedings of a conference held
at the National Bureau of Standards,
Gaithersburg, MD, April 6-10, 1981

FOREWORD

The study of fluctuations (or noise) in a physical system provides insights, not available by any other technique, into the microscopic dynamic behavior of that system. Besides being a source of information, noise can also be a source of irritation, in that it limits the performance of numerous devices. The study of noise is of prime importance for the testing of physical theories as well as for the development of improved physical measurements and improved performance of devices. Therefore, the Conference has as one of its goals an improved understanding of noise in devices and its influence on the error budget of a measurement. Indeed, progress in relieving or minimizing noise in some devices was reported (e.g., the relationship of "burst noise" to the metallurgical condition of the sample).

Strong emphasis was given in this Conference to new topics for which the noise spectra proved to be particularly helpful in characterizing the underlying system dynamics. Papers discussed, for example, the transition from periodic to chaotic behavior in chemical systems and turbulent fluid flow, entropy generation in the computer process, the existence and implications of quantum mechanical noise, and noise spectra occurring in electrochemical processes.

Judging from the number of contributions and the intensity of the discussions following their presentations, the topic of 1/f noise remains as a very interesting one. It has resisted most, if not all theoretical attempts to explain it. An invited paper by T. Musha gave even more evidence to its ubiquity in nature. One of the most interesting developments here has been the connection between 1/f noise and human comfort. Extending beyond the observation that noise exhibiting a 1/f spectrum is pleasing to the listener, clinical evidence now suggests that electronic alleviation of pain in humans is improved when the electrical shocks are given a 1/f component.

Scaling phenomena: Time scales

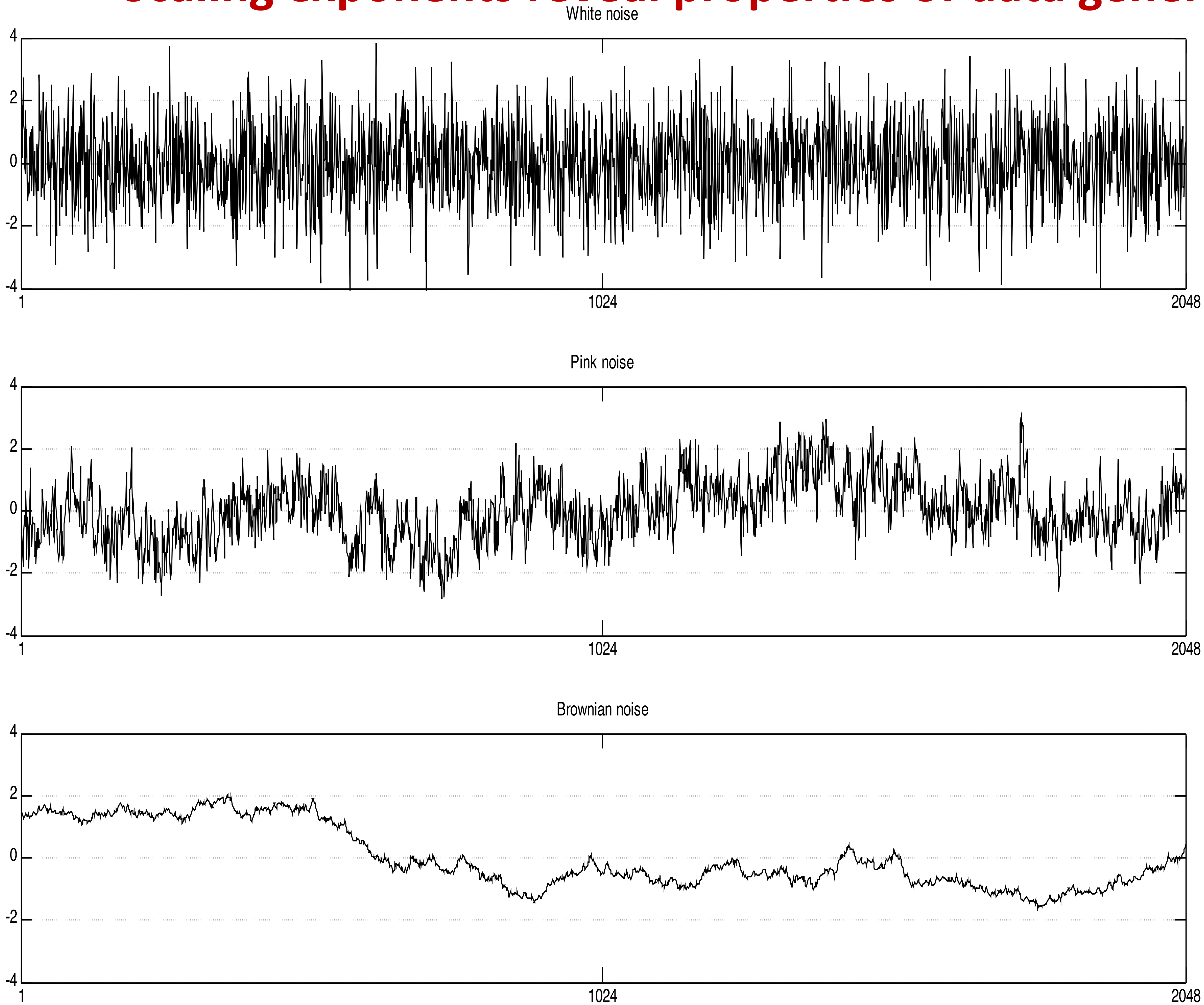
1/*f* Noise in Human Cognition

D. L. Gilden,* T. Thornton, M. W. Mallon

When a person attempts to produce from memory a given spatial or temporal interval, there is inevitably some error associated with the estimate. The time course of this error was measured in a series of experiments where subjects repeatedly attempted to replicate given target intervals. Sequences of the errors in both spatial and temporal replications were found to fluctuate as 1/*f* noises. 1/*f* noise is encountered in a wide variety of physical systems and is theorized to be a characteristic signature of complexity.

SCIENCE • VOL. 267 • 24 MARCH 1995

Scaling exponents reveal properties of data generating processes



White noise ~ 1.5

RANDOM
UNCORRELATED
UNCONSTRAINED

Pink noise ~ 1.2

Between:
order - random
constrained - unconstrained

Long-range dependence
Self-Organised Criticality

PERSISTENT
HIGHLY CORRELATED
CONSTRAINED

Random walk ~ 1.1

Line–like ~ 1

Fractal Physiology

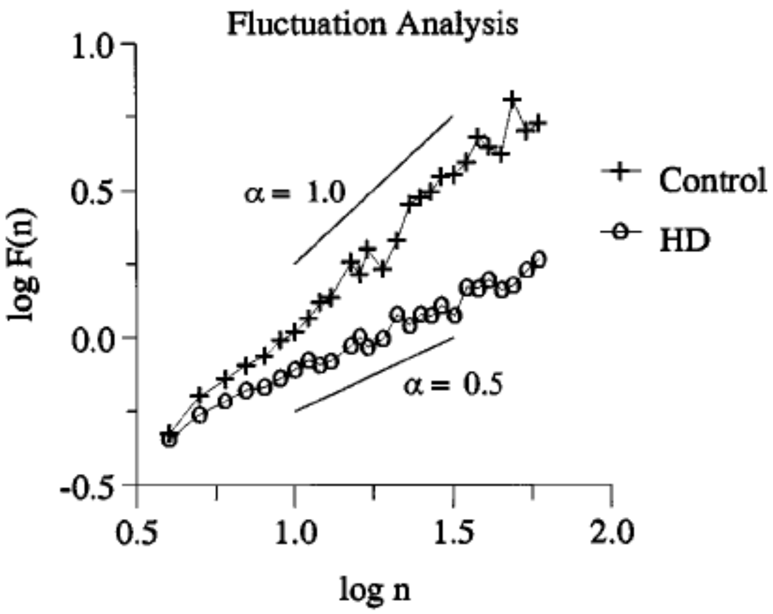
Multiplicative cascade / Multifractal formalism

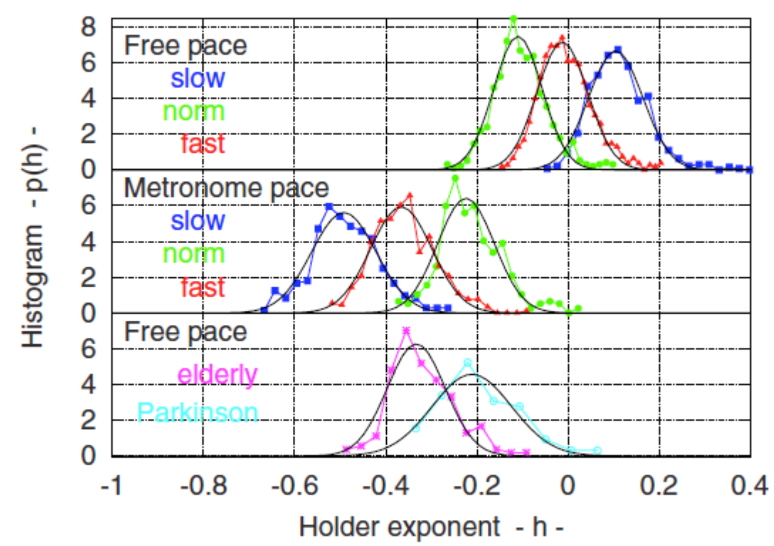
Monofractal

Multifractal

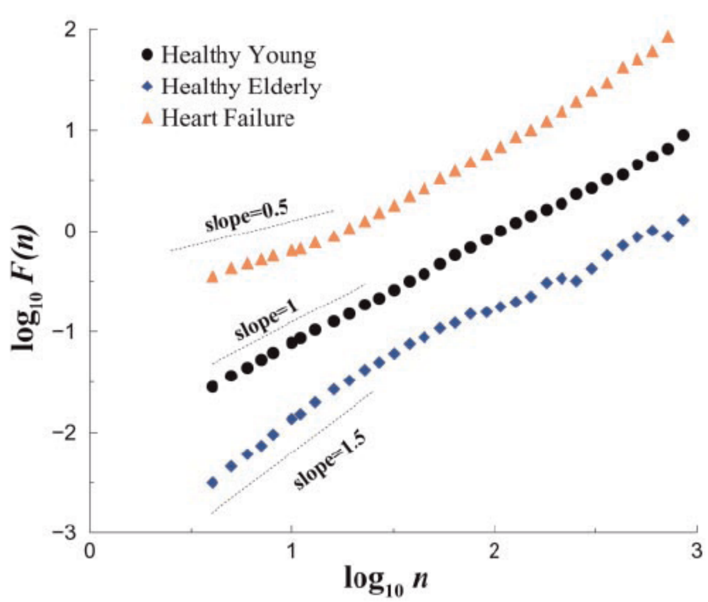
Attraction to criticality

Stride intervals





Heart rate variability



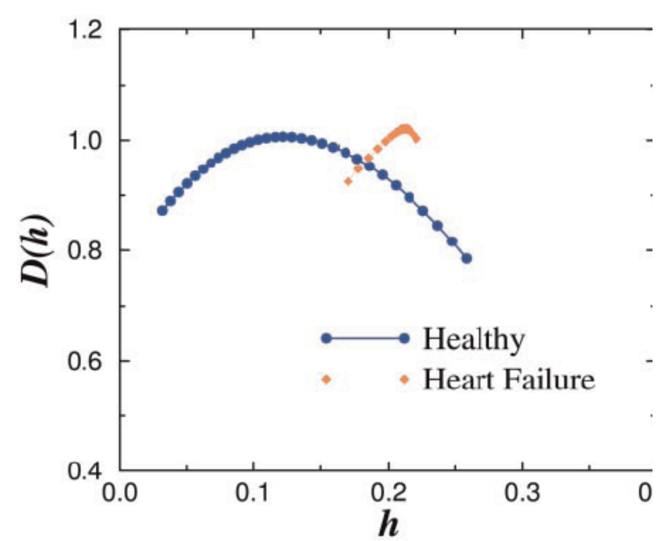
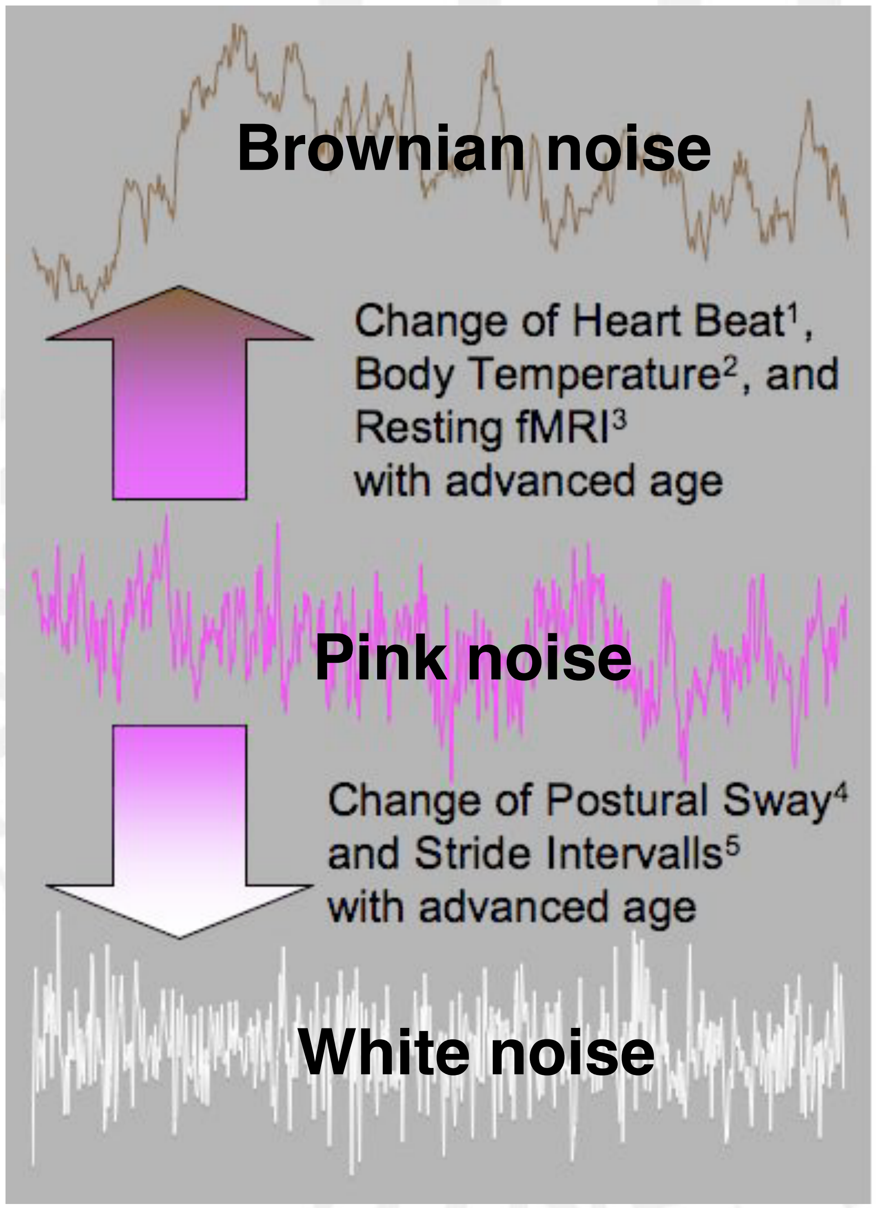


Fig. 7. Singularity spectra of heart rate signals in health and disease function *D(h)* measures the fractal dimension of the subset of the signal t



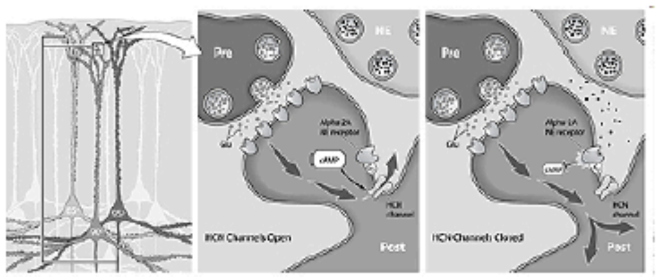
INTERVENTION: Almurad, Z. M., Roume, C., Blain, H., & Delignières, D. (2018). Complexity matching: Restoring the complexity of locomotion in older people through arm-in-arm walking. *Frontiers in physiology*, *9*, 1766.

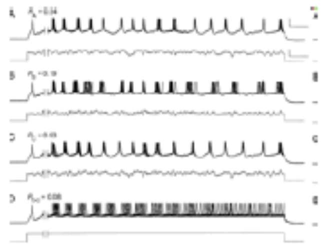
Fractal Neurophysiology

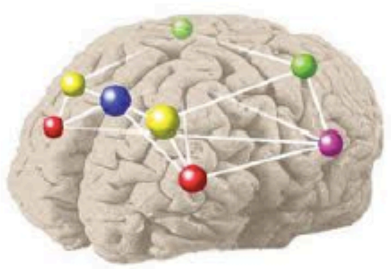
1/f noise in the Brain

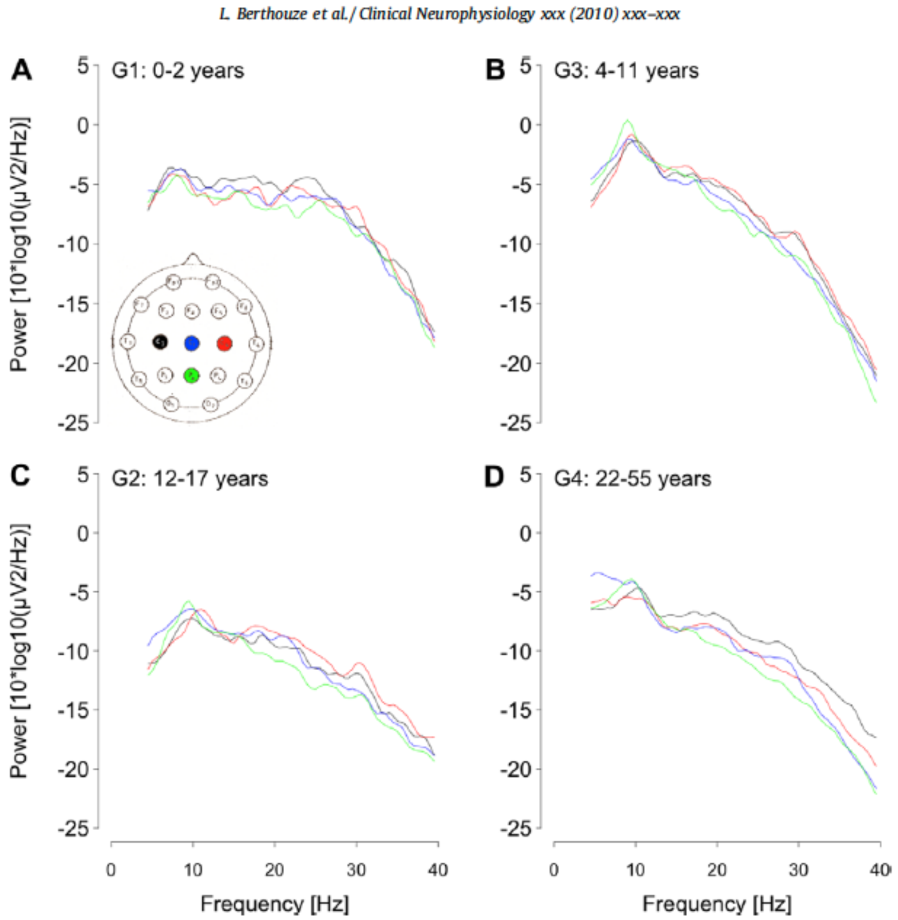
Wijnants, M. (2011)

- Ion Channels Opening and Closing Times
  - (Liebovitch & Krekora, 2002; Liebovitch & Shehadeh, 2005; Lowen , Cash, Poo, & Teich, 1997; Takeda, Sakata, & Matsuoka, 1999, Varanda, Liebovitch, Figueiroa, & Nogueira, 2000)
- Neural Spike Intervals
  - (Bhattacharya, Edwards, Mamelak, & Schuman, 2005; Giugliano, Darbon, Arsiero, Luescher, & Streit, 2004; Grüneis et al., 1993, West & Deering, 1994)
- Larger Scale Neural Assemblies
  - (Buzsàki, 2006; Bressler & Kelso, 2001; Freeman, Holmes, Burke, & Vanhatalo, 2003; Spasic, Kesic, Kalauzi, & Saponjic, 2010; Tognoli & Kelso, 2009; Varela, Lachaux, Rodriguez, & Martinerie, 2001; Werner, 2007)



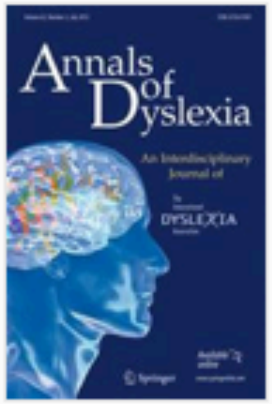






Berthouze, L., James, L. M., & Farmer, S. F. (2010). Human EEG shows long-range temporal correlations of oscillation amplitude in Theta, Alpha and Beta bands across a wide age range. Clinical neurophysiology, 121(8), 1187-97. doi: 10.1016/j.clinph.2010.02.163.

Radboud University Nijmegen



Annals of Dyslexia
July 2012, Volume 62, Issue 2, pp 100–119 | Cite as

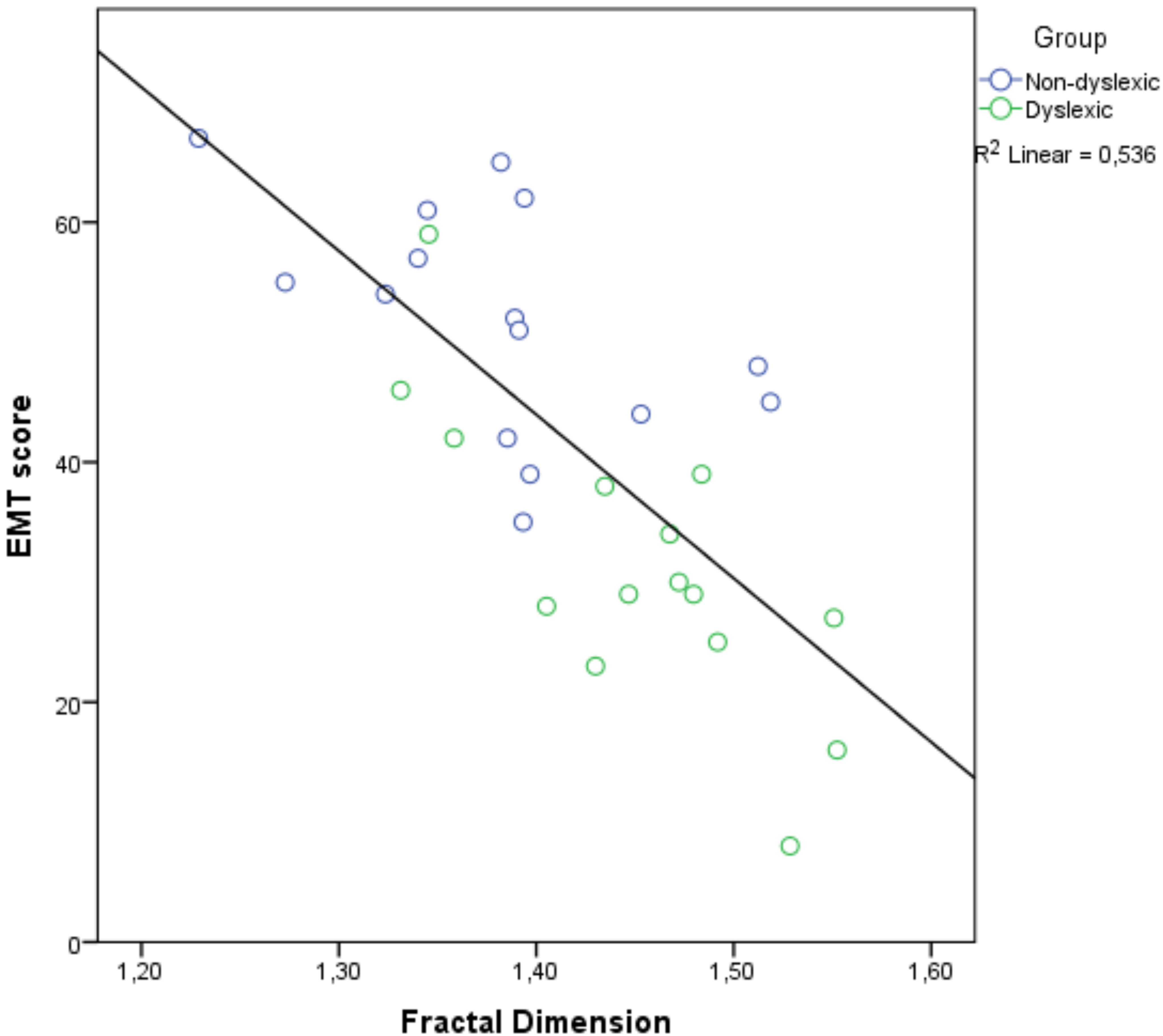
An interaction-dominant perspective on reading fluency and dyslexia

Authors | Authors and affiliations

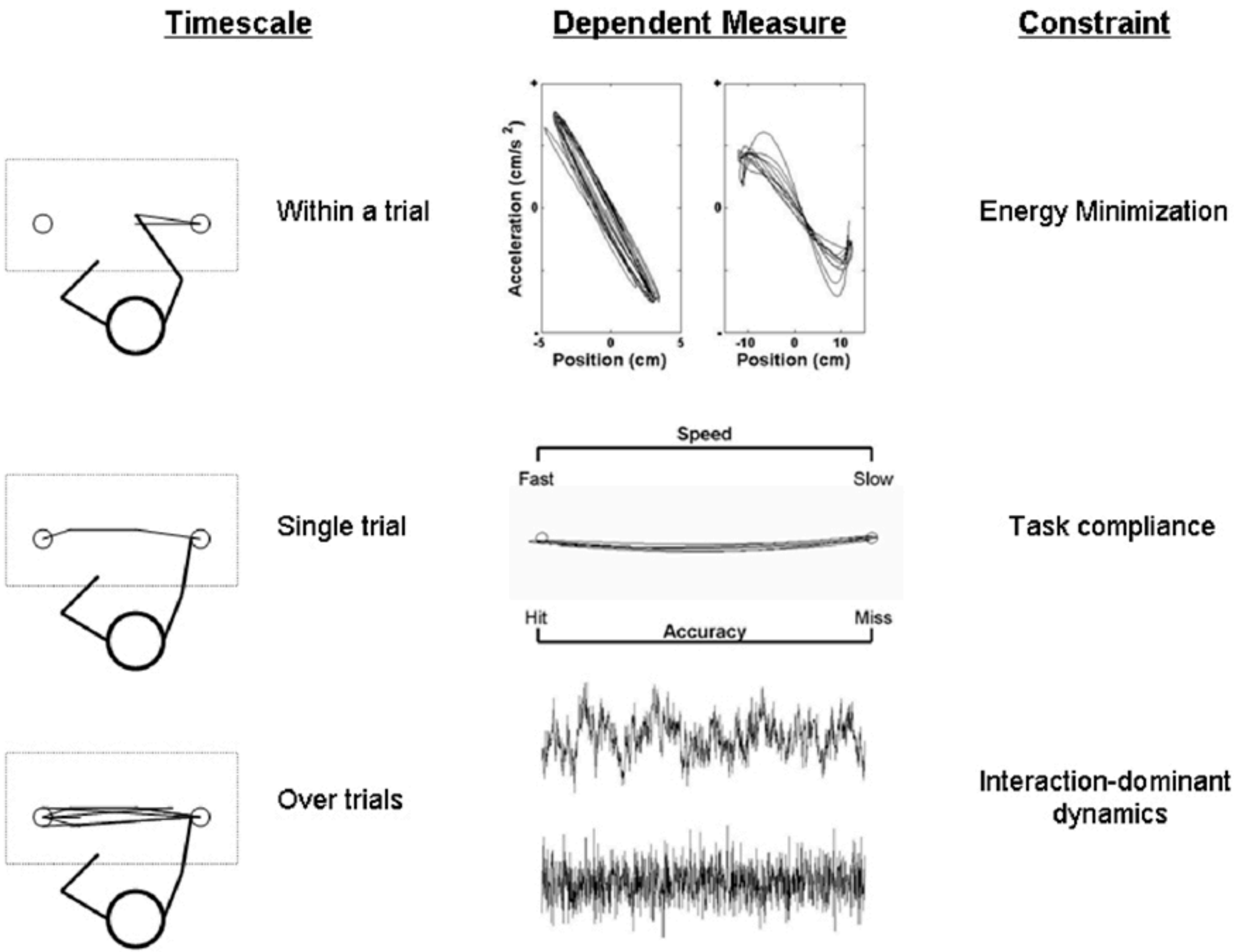
M. L. Wijnants, F. Hasselman, R. F. A. Cox, A. M. T. Bosman, G. Van Orden

Open Access | Article
First Online: 30 March 2012

- 560 single-syllable words
- Fast + accurate
- Record naming latency



Group
Non-dyslexic
Dyslexic
R2 Linear = 0,536
EMT score
0
20
40
60
1,20
1,30
1,40
1,50
1,60
Fractal Dimension



Wijnants, M., Cox, R., Hasselman, F., Bosman, A., & Van Orden, G. (2012). A trade-off study revealing nested timescales of constraint. *Frontiers in physiology*, *3*, 116.

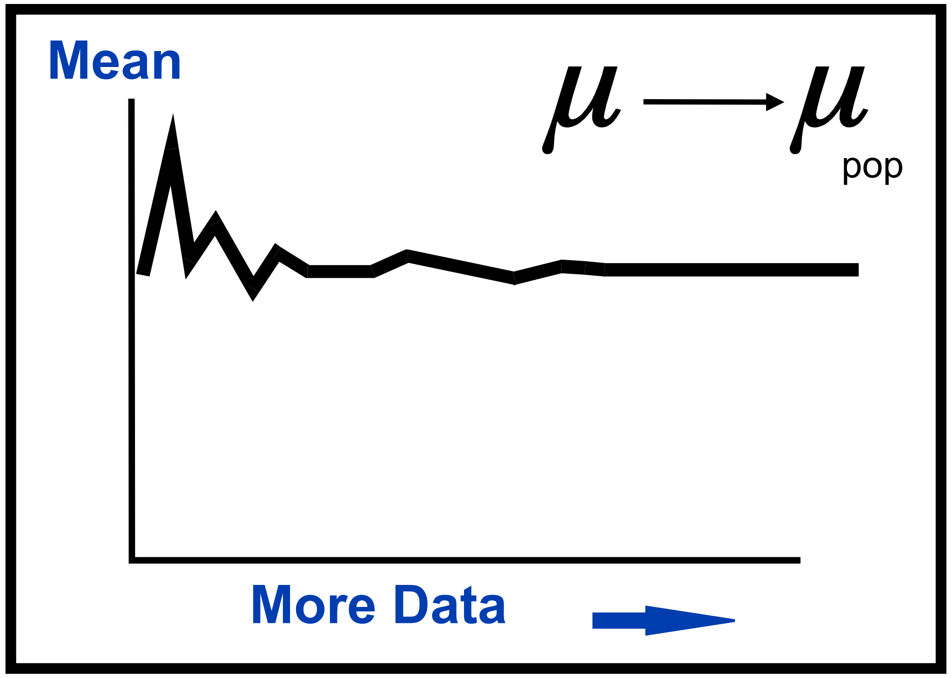
Experimental Control over Scaling >> applications in sports science, e.g. cycling, rowing, swimming

- Hoos O., Boeselt T., Steiner M., Hottenrott K., Beneke R. (2014). Long-range correlations and complex regulation of pacing in long-distance road racing. Int. J. Sports Physiol. Perform. 9, 544–553. 10.1123/ijspp.2012-0334.
- Den Hartigh, R. J., Cox, R. F., Gernigon, C., Van Yperen, N. W., & Van Geert, P. L. (2015). Pink noise in rowing ergometer performance and the role of skill level. Motor control, 19(4), 355-369.
- Nourrit-Lucas, D., Tossa, A. O., Zélic, G., & Delignières, D. (2015). Learning, motor skill, and long-range correlations. Journal of motor behavior, 47(3), 182-189.
- Barbosa, T. M., Goh, W. X., Morais, J. E., Costa, M. J., & Pendergast, D. (2016). Comparison of classical kinematics, entropy, and fractal properties as measures of complexity of the motor system in swimming. Frontiers in psychology, 7, 1566.
- Den Hartigh, R. J., Marmelat, V., & Cox, R. F. (2018). Multiscale coordination between athletes: Complexity matching in ergometer rowing. Human movement science, 57, 434-441

Scaling phenomena: Time scales

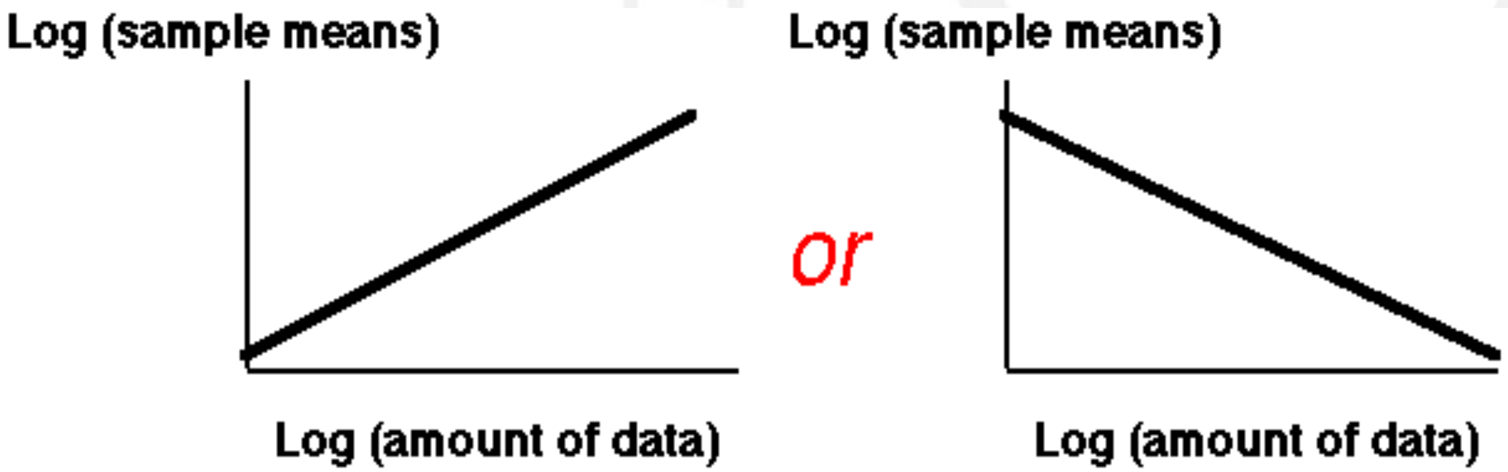
Independent observations of random variables

μ±σ are sufficient to characterise absence of dependencies in the data:
e.g. Expected value of **μ** for **N** = 100, given **σ**
N = ensemble size

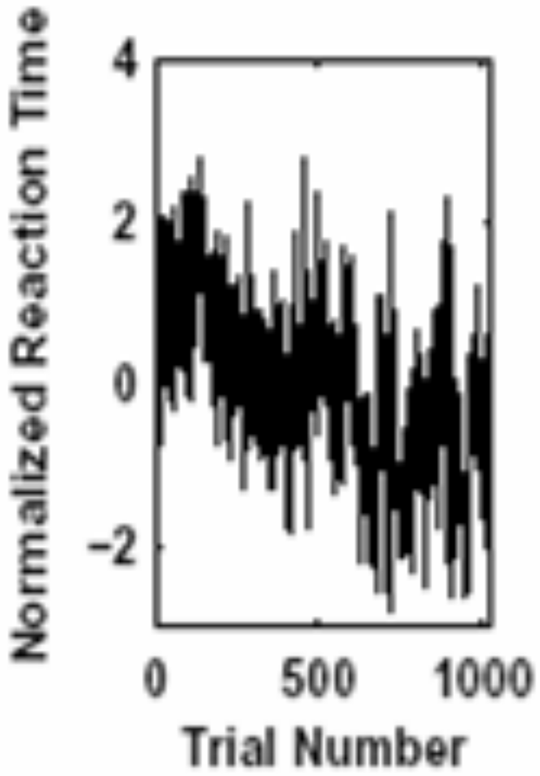


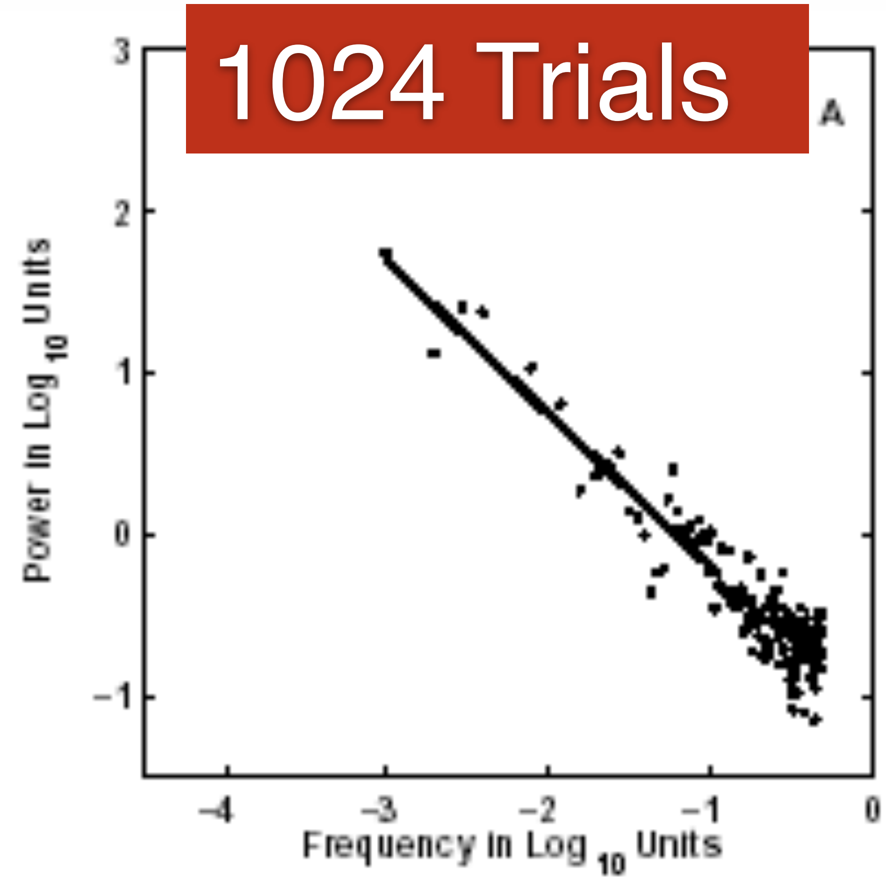
Interdependent observations across different scales

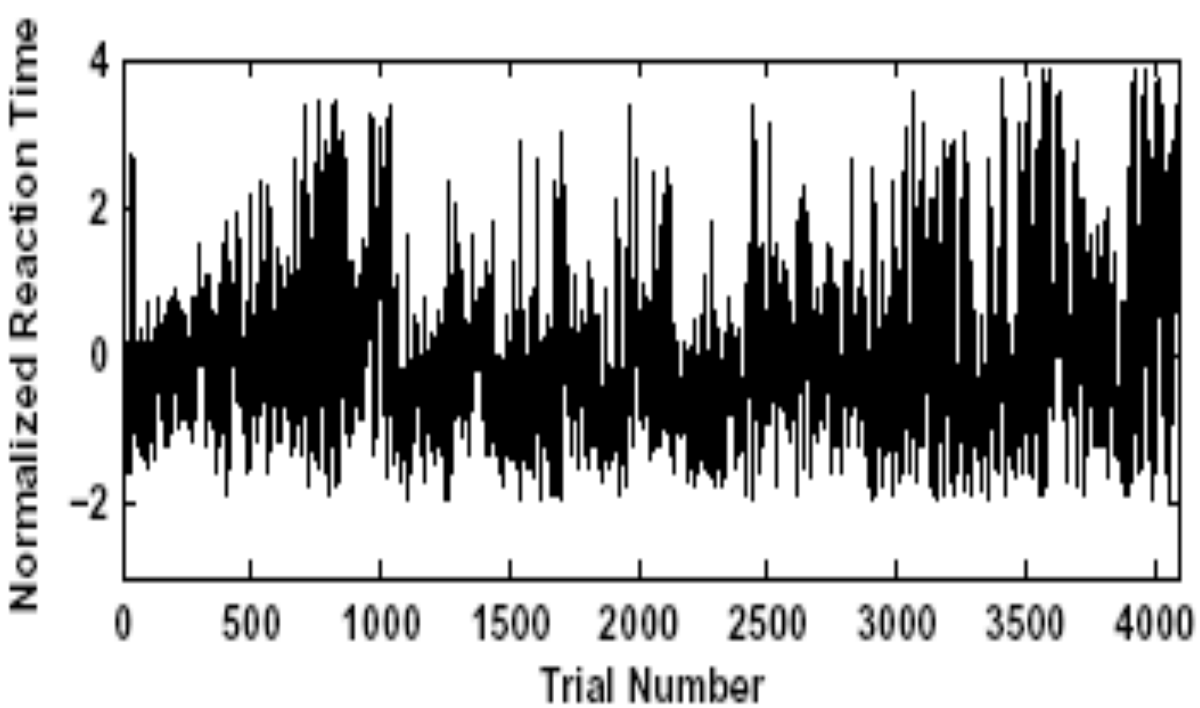
μ±σ are insufficient to characterise dependencies in the data:
e.g. Sample estimates of **μ** change with **N**
N = observation time

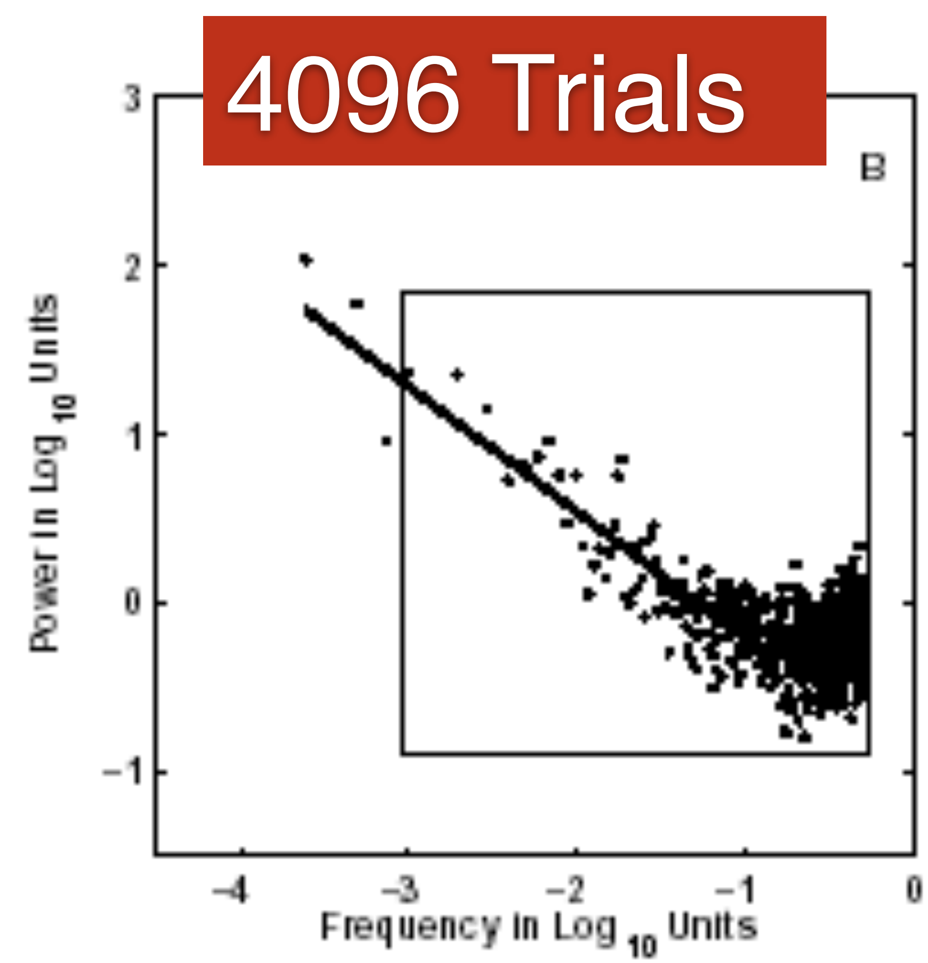


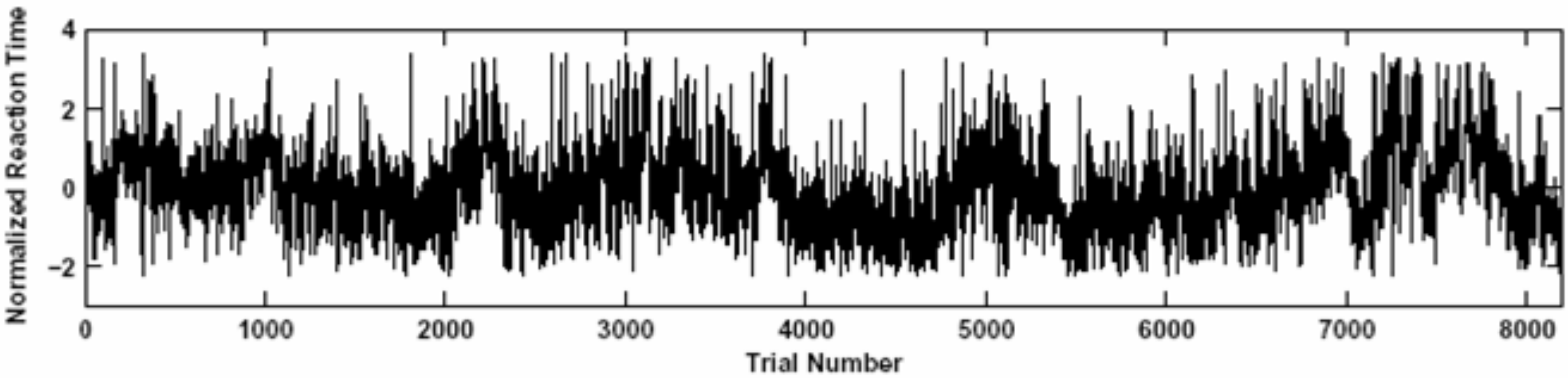
“Statistics”: More data = more variance

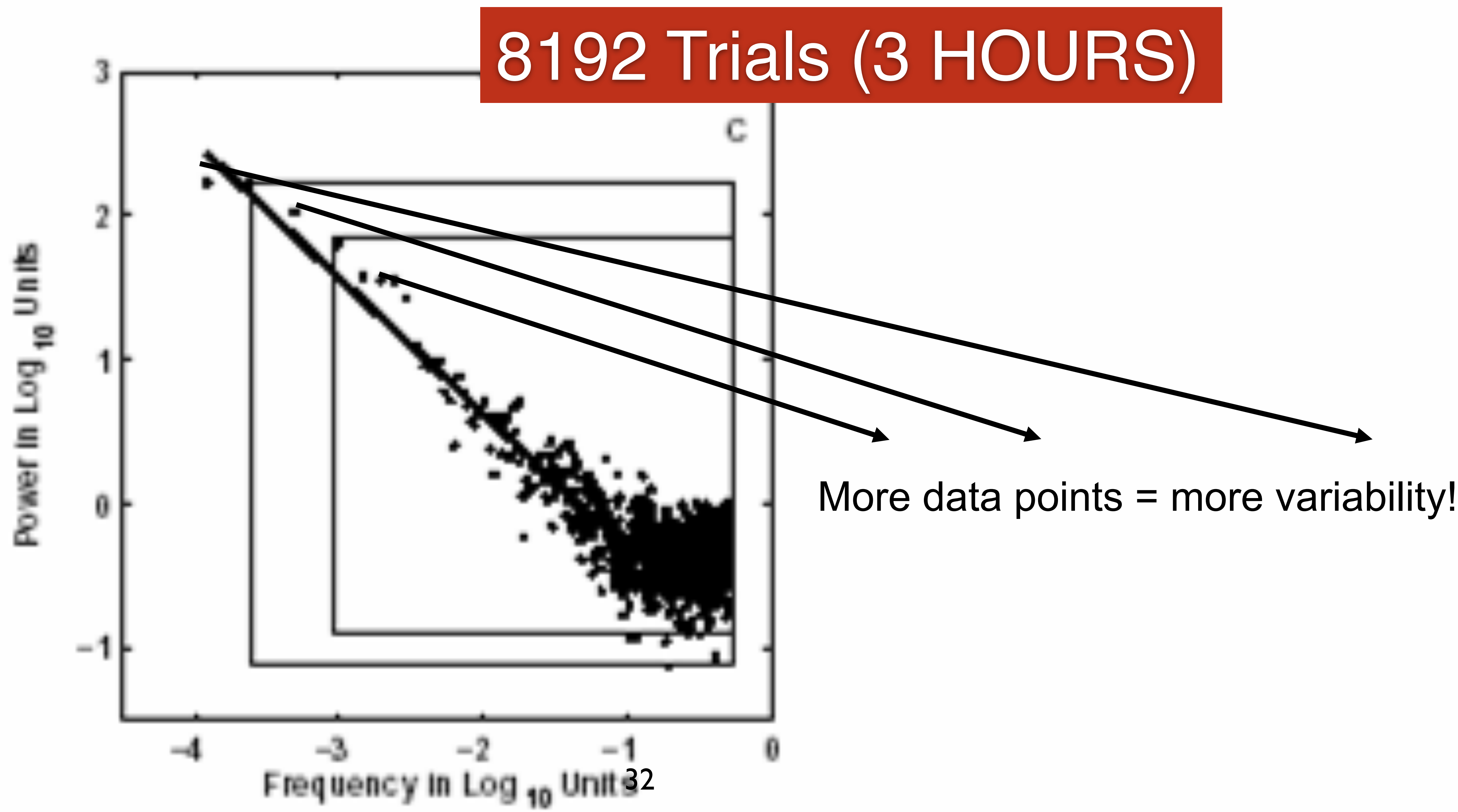








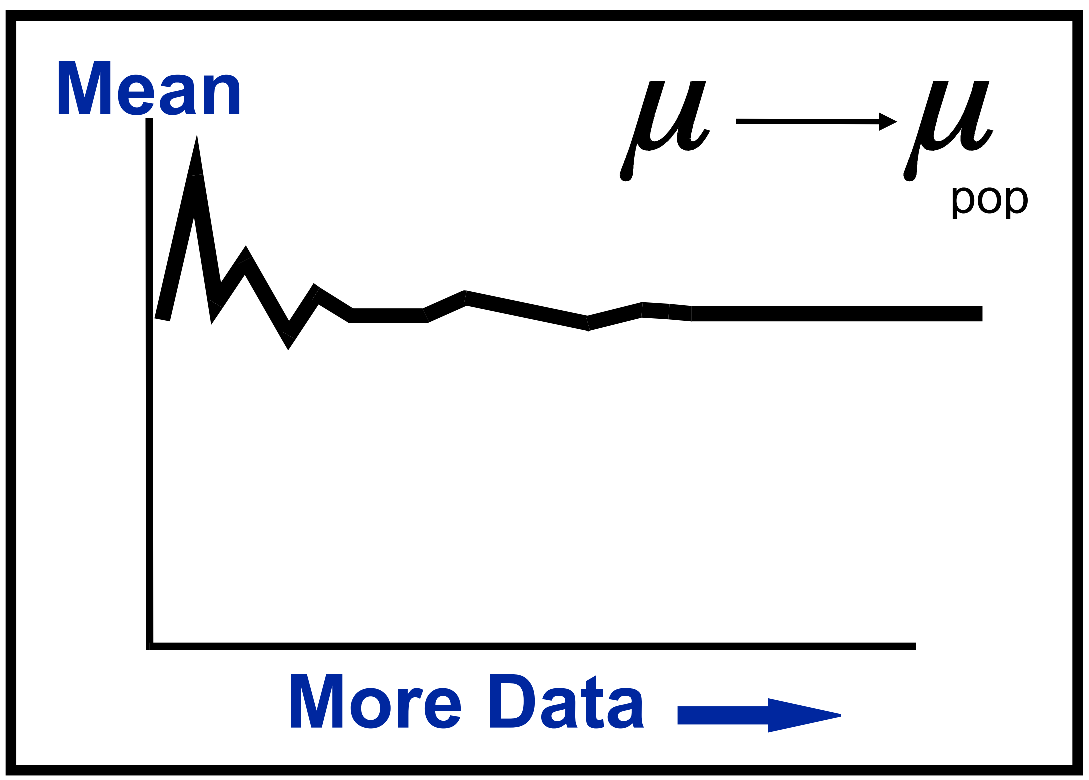

Normalized Reaction Time
4
2
0
−2
0
1000
2000
3000
4000
5000
6000
7000
8000
Trial Number


8192 Trials (3 HOURS)
C
3
2
1
0
−1
−4
−3
−2
−1
0
Power in Log 10 Units
Frequency in Log 10 Units
More data points = more variability!

"Statistics": Variance will not decrease with more observations!!!

Random processes
Random variables

Independent observations
(no-similarity = random)
Characteristic scale: **T**
(the population)



Fractal processes
Fractal variables

Interdependent observations over different scales!
(self-similarity = "correlated")
No characteristic scale means: **T** does not exist!
(at least not on 1 scale)