About the course….

Course site: https://complexity-methods.github.io

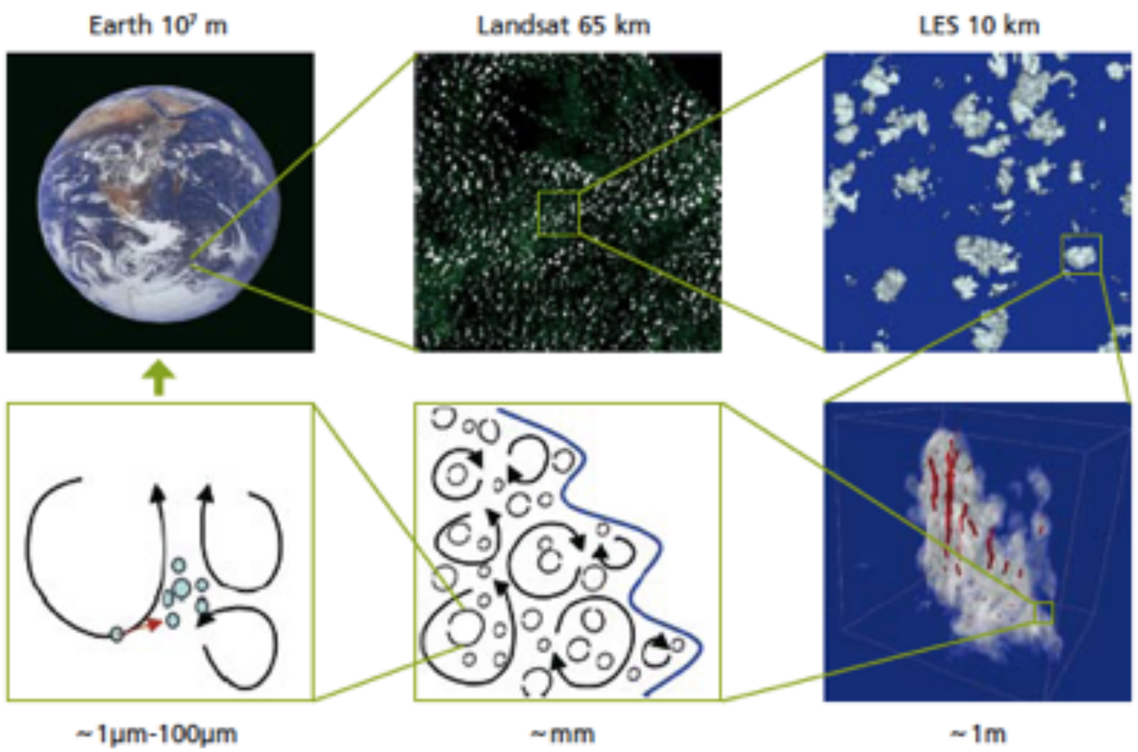
Complexity Methods for Behavioural Science

Day 1: Intro to Complexity Science

Intro Mathematics of Change
Basic Timeseries Analysis
Basic Nonlinear Timeseries Analysis
Scaling

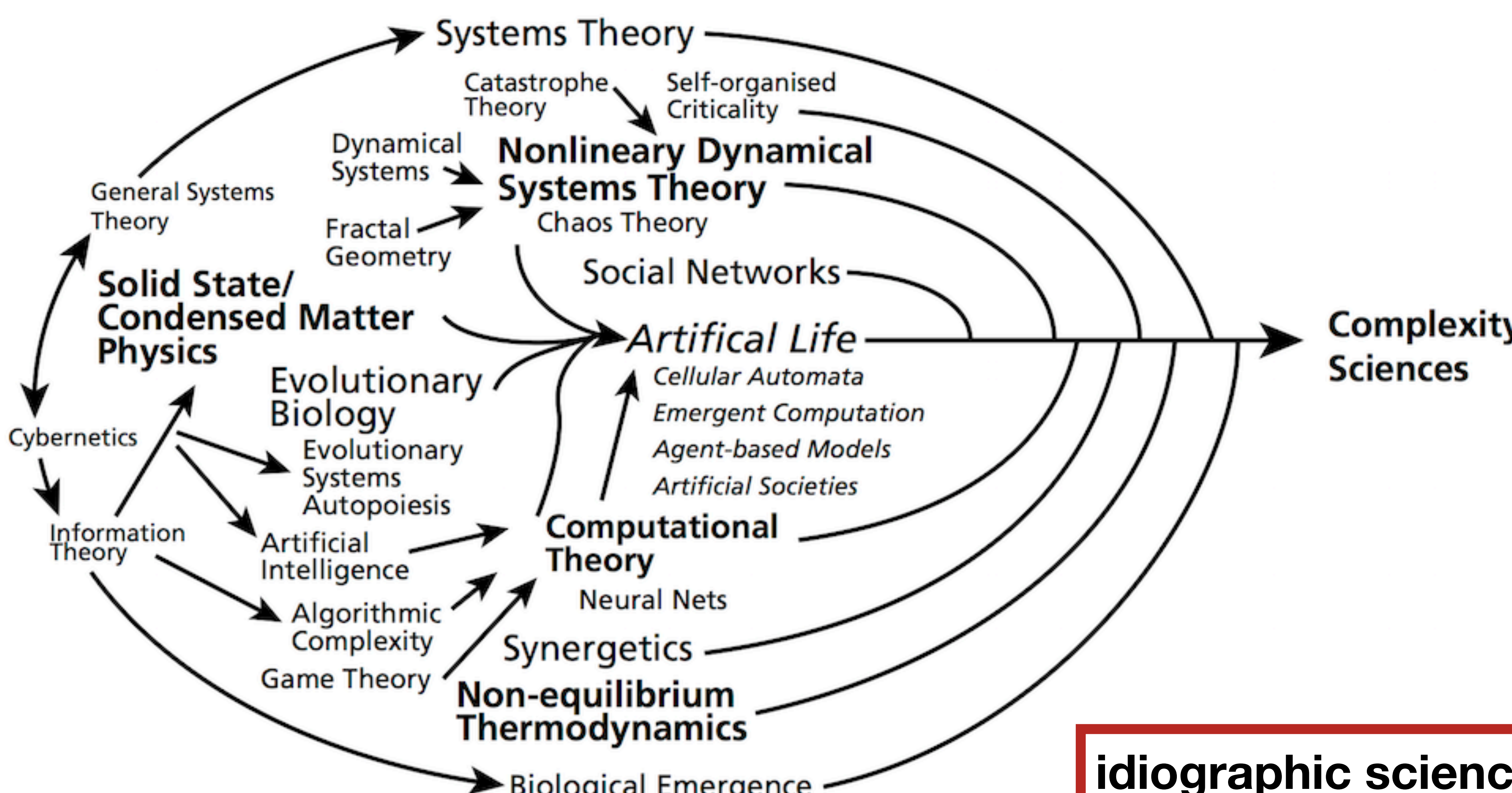
Complexity Science

- Time! (Dynamics)
- Micro-Macro levels (Emergence)
- Self-Organization
- Scale invariance



Complexity Science

The scientific study of complex dynamical systems and networks



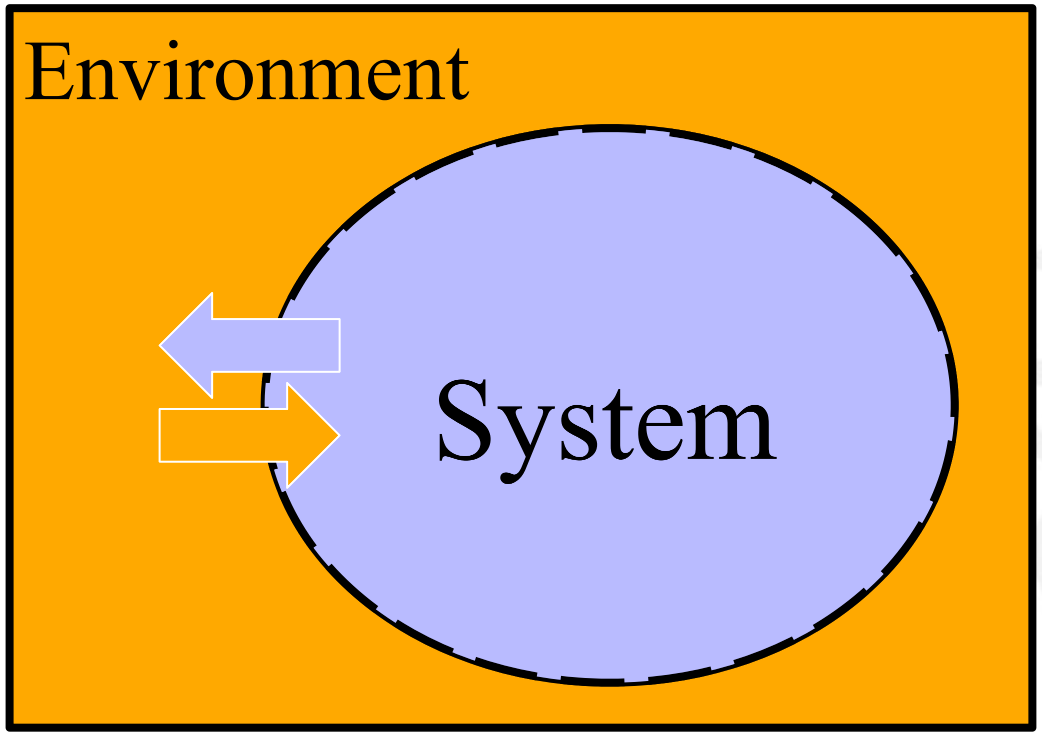
idiographic science!

The many foundations of complexity science.

What is a system?

A system is an entity that can be described as a composition of components, according to one or more organising principles.

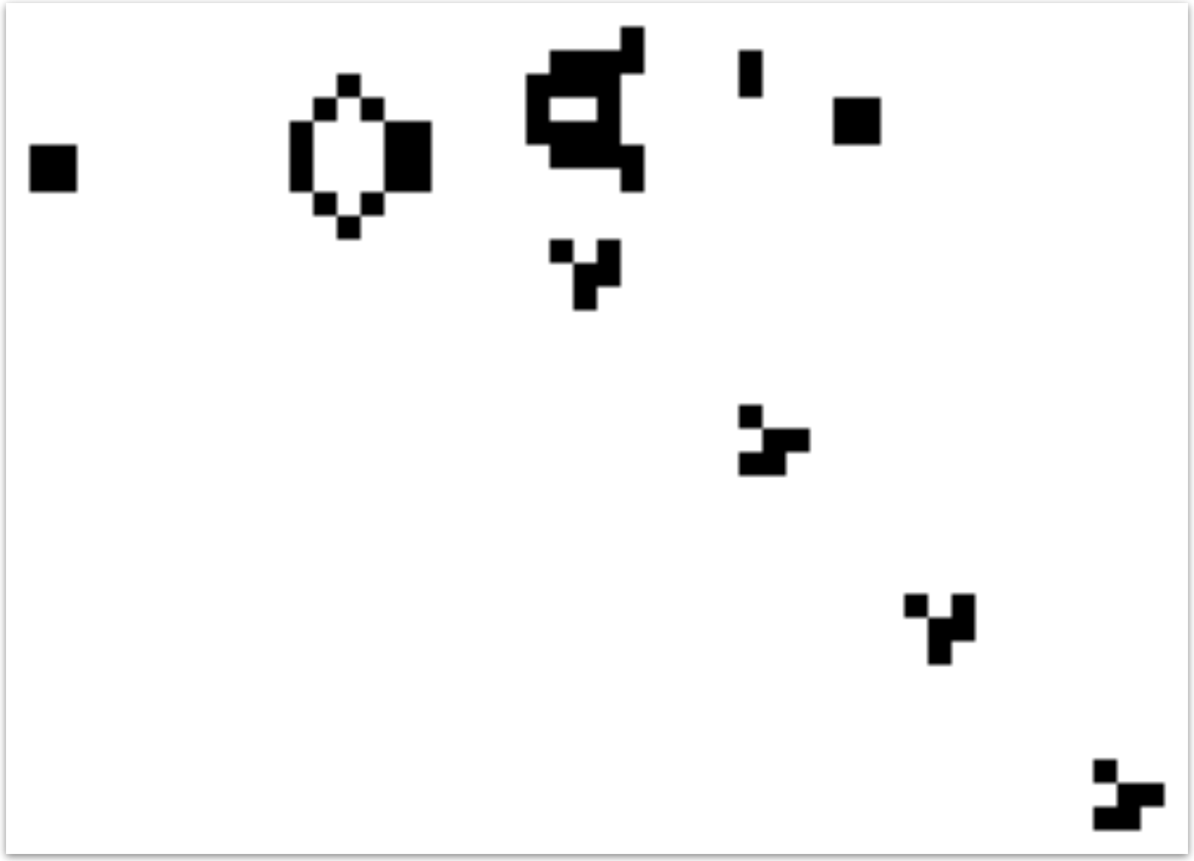
Closed and Open Systems



Continuous exchange of matter, energy, and information with the environment.

MICRO-MACRO levels
Emergent patterns... swarms, schools

Glider gun creating "Gliders"



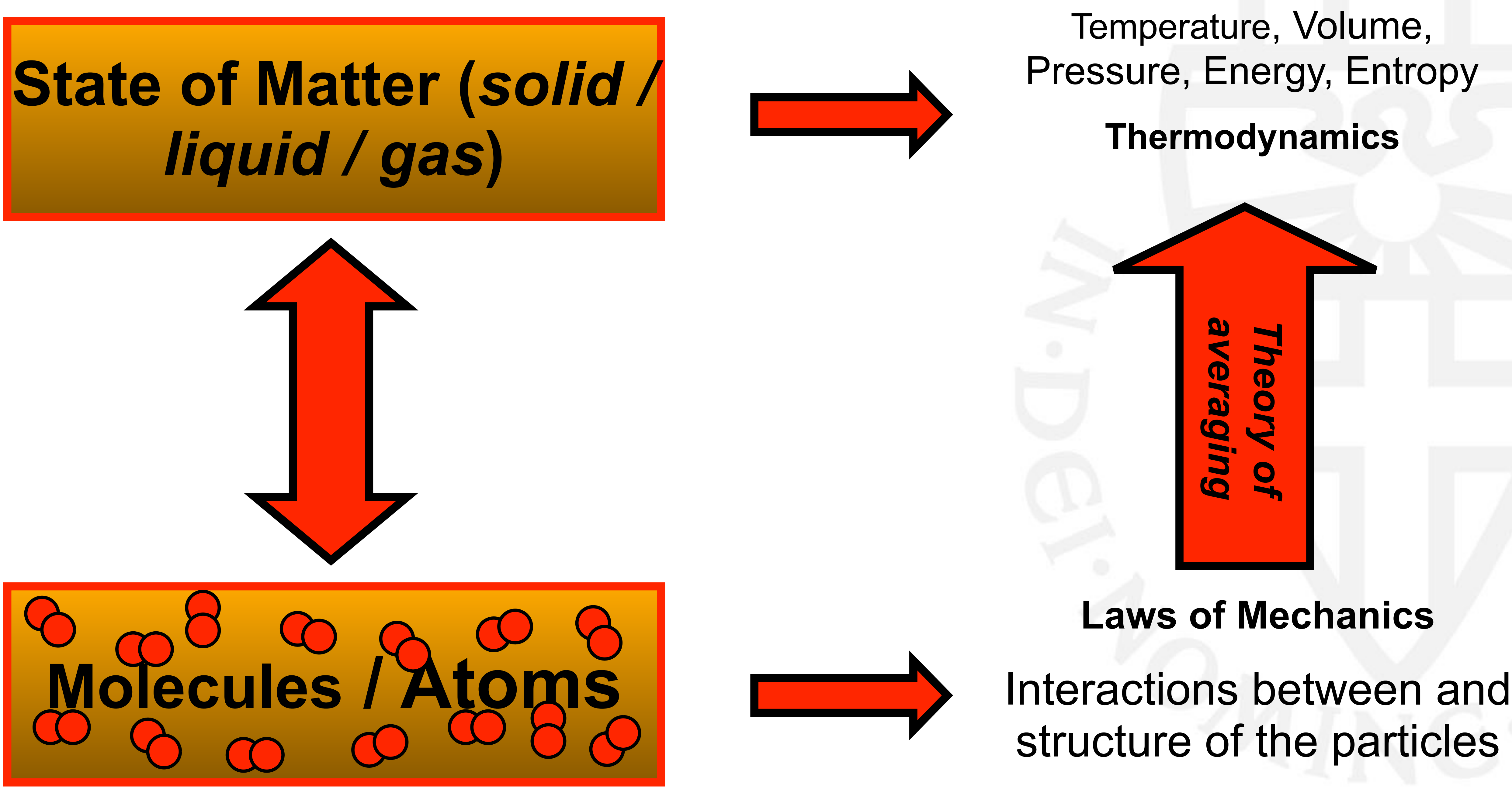
http://en.wikipedia.org/wiki/Gun_(cellular_automaton)

http://www.google.com/imgres?imgurl=http://www.projects-abroad.org/_photos/_global/photo-galleries/en-uk/cambodia/_global/large/school-of-fish.jpg&imgrefurl=http://www.projects-abroad.org/photo-galleries/?content=cambodia/&usg=__xPQQdvCtelyjDbZZu79223c58A

Levels of Analysis: Micro - Macro

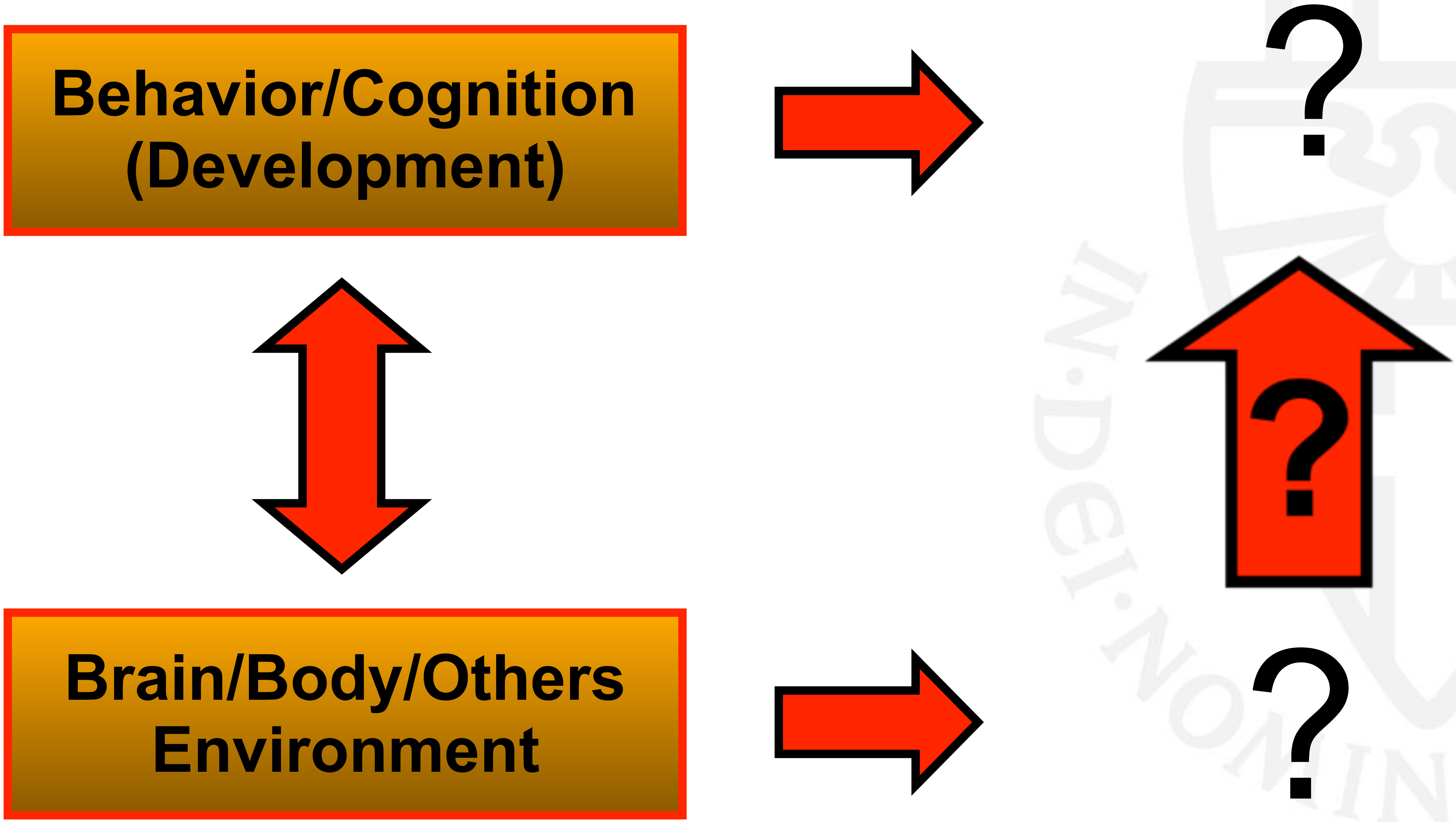
Forms and properties
are emergent,
not expected from
components:
1 watermolecule
does not possess the
property "wet"

Levels of Analysis: Micro - Macro

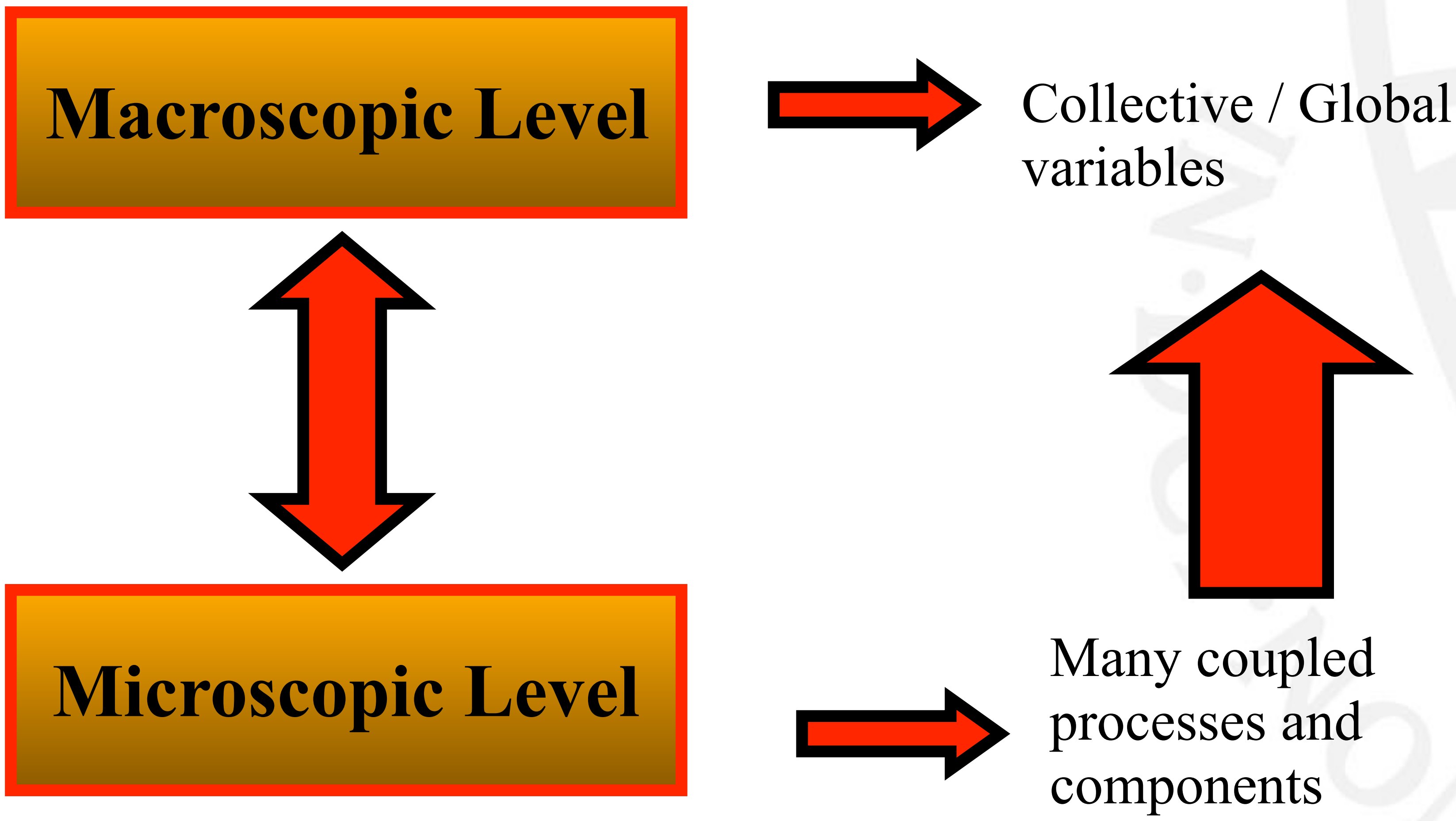


Levels of Analysis: Micro - Macro

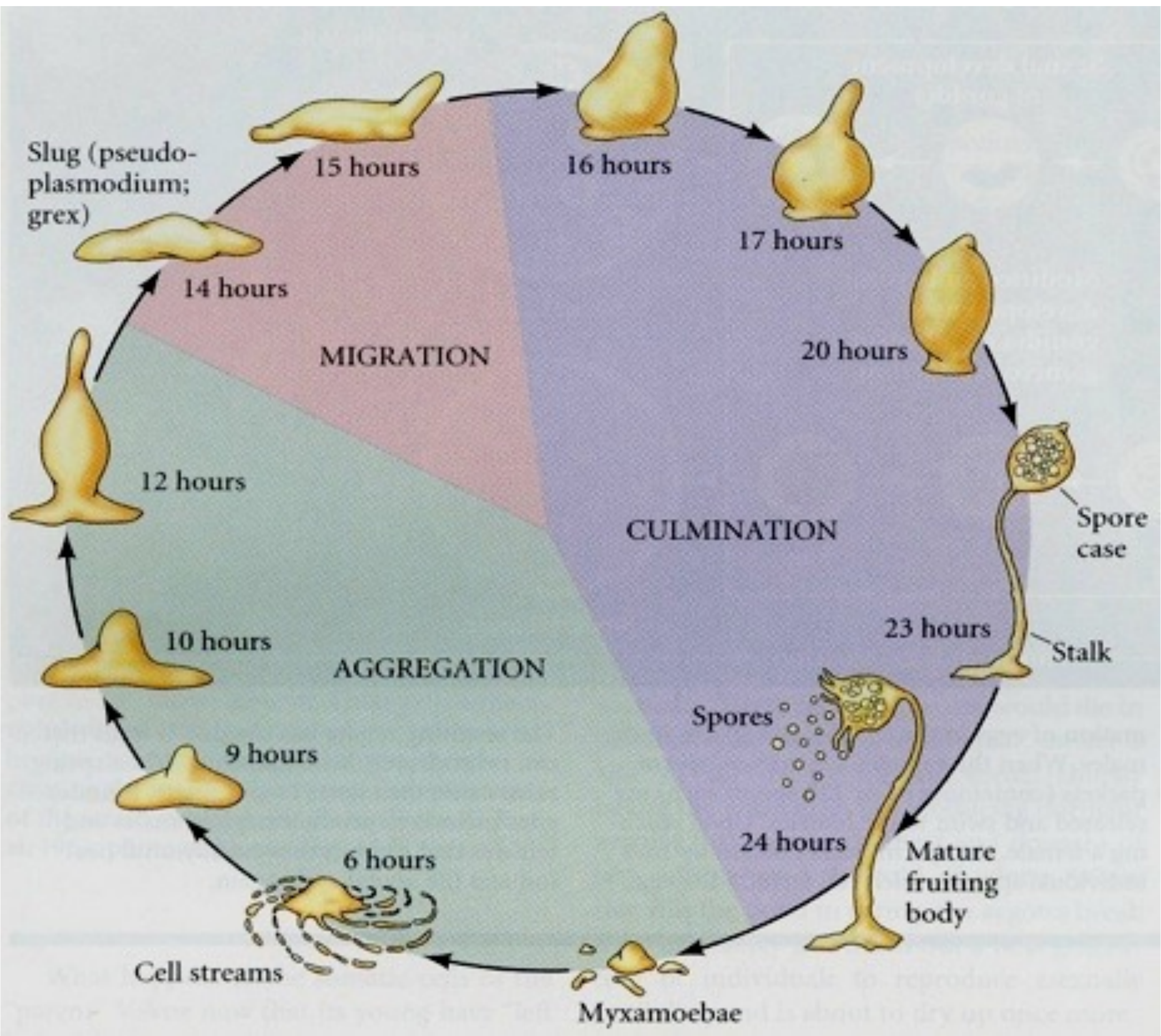
Much to be filled in!



Levels of Analysis: Micro - Macro

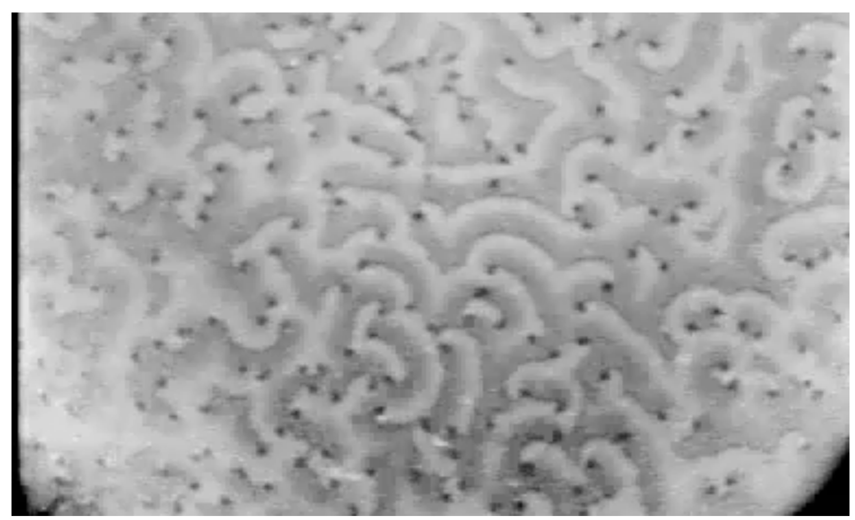


Emergence and Self-Organization: The life-cycle of *Dictyostelium*



1. Free living myxamoebae feed on bacteria and divide by fission.
2. When food is exhausted they aggregate to form a mound, then a multicellular slug.
3. Slug migrates towards heat and light.
4. Differentiation then ensues forming a fruiting body, containing spores.
5. It all takes just 24 hrs.
6. Released spores form new amoebae.

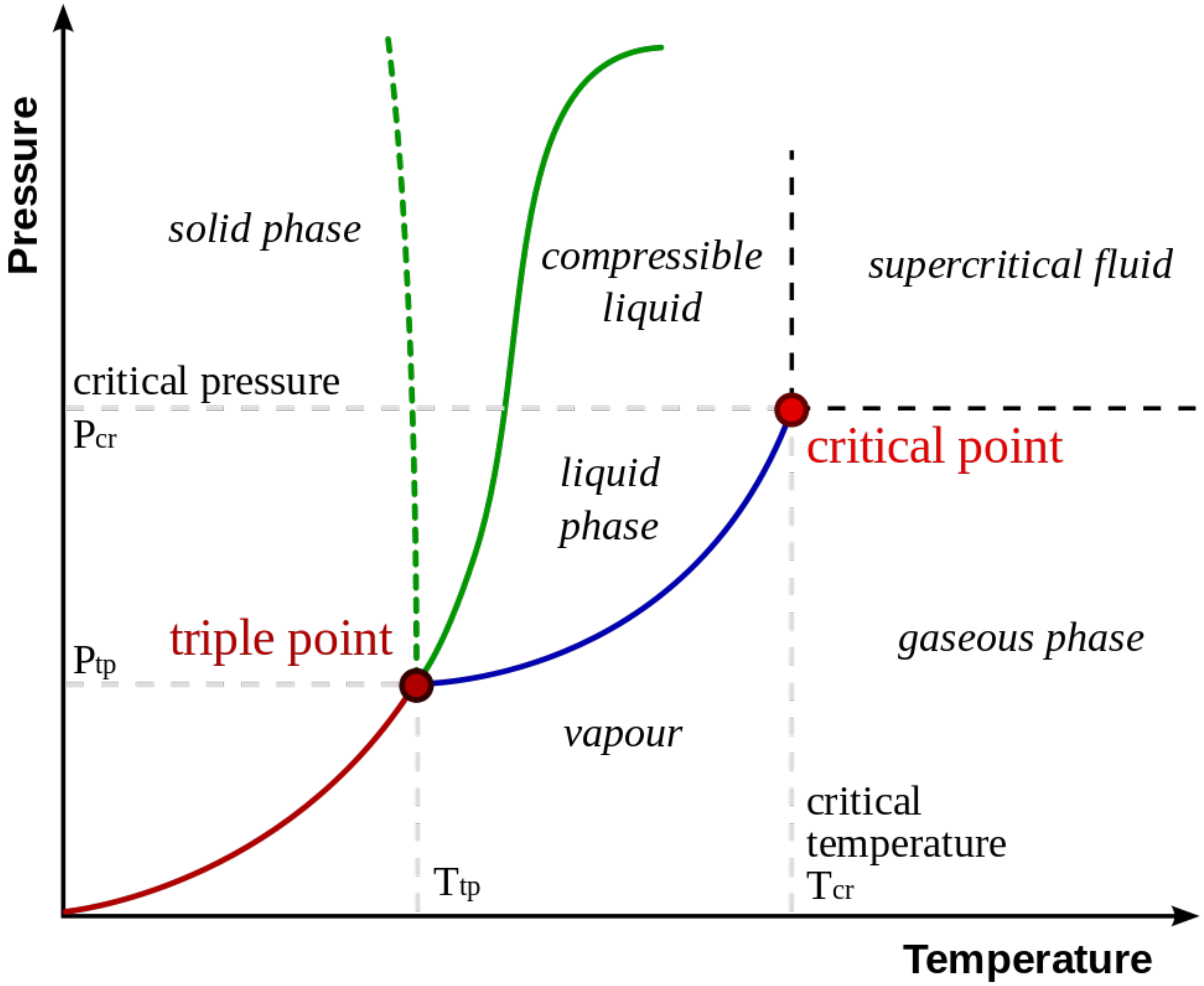
Order parameter: Labelling states of a complex system

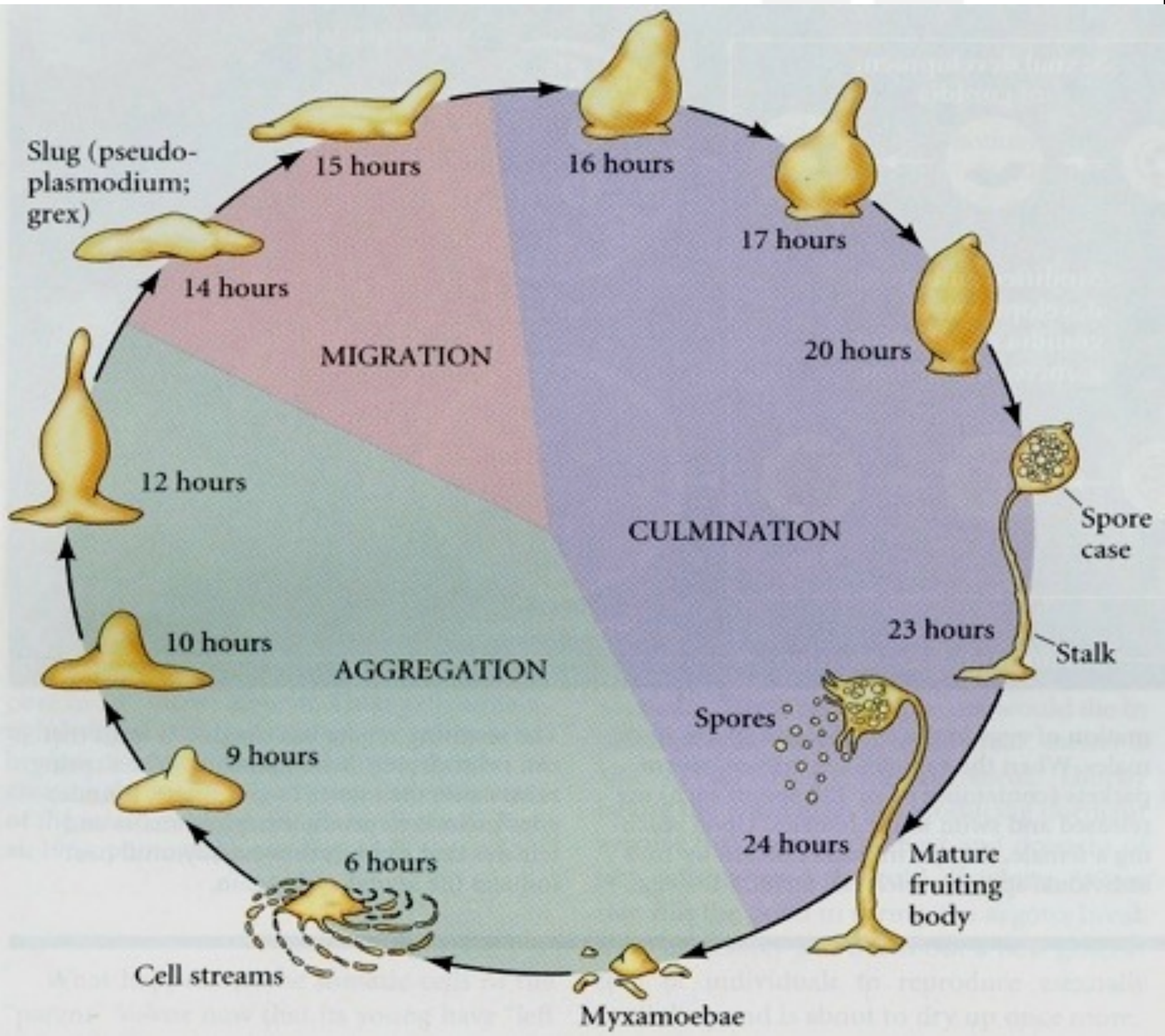


Forms are emergent, self-organised:

Arise from interactions between components → reduction of degrees of freedom

Phase Diagram & Order parameter





The order parameter is often a qualitative description of a macro state / global organisation of the system, conditional on the control parameters:

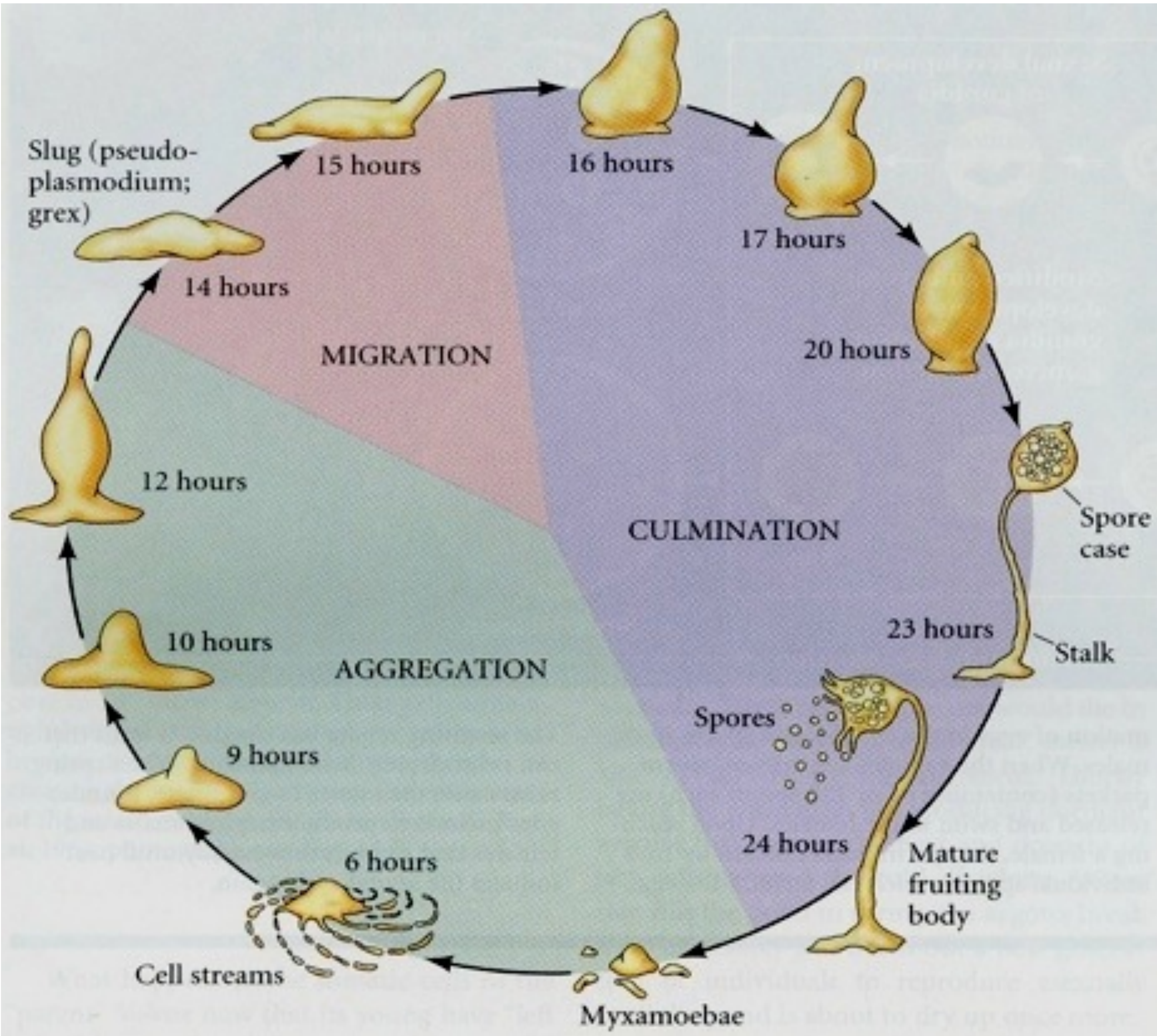
H_2O: Ice (Solid), Water (Liquid), Steam (Vapour)

Disctyostelium: Aggregation (Mound), Migration (Slug), Culmination (Fruiting Body)

https://youtu.be/Juz9pVVsmQQ

Dynamic Metaphor vs. Dynamic Measure

Metaphor: Sate Space / Order Parameter
Measures: Attractor strength / Stability



Order parameter: the qualitatively different states

Control parameter: available food (actually concentration of a chemical that is released if they are starving)

Experiments:
Find out if the process is reversible... add food

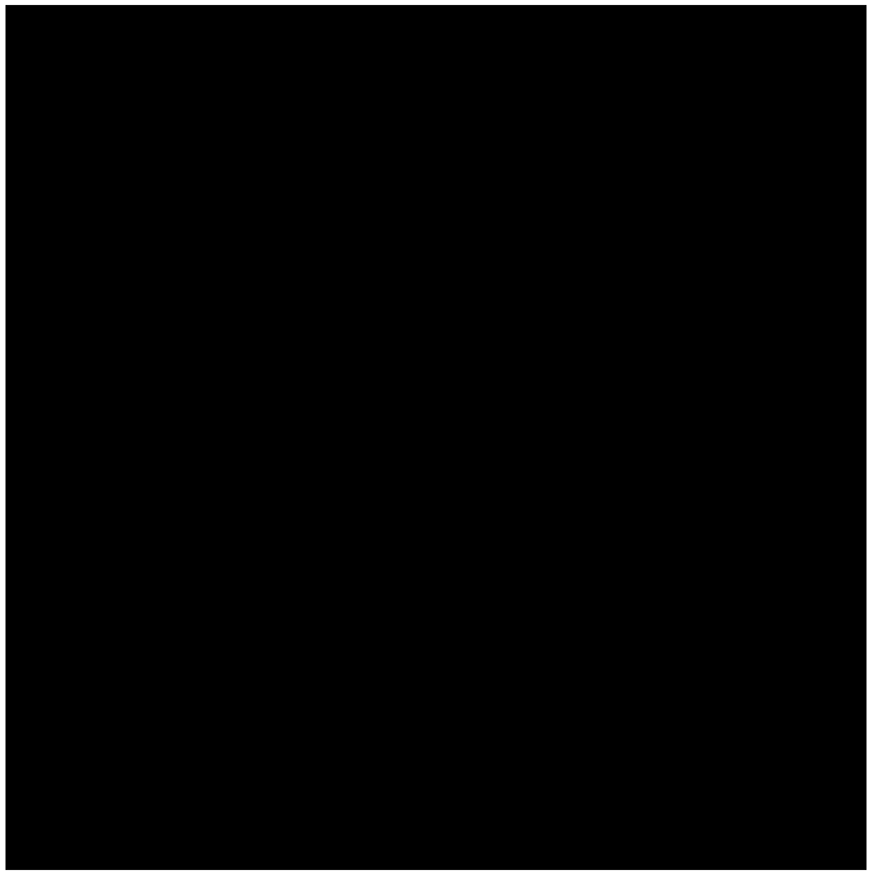
perturb the system during the various phases...

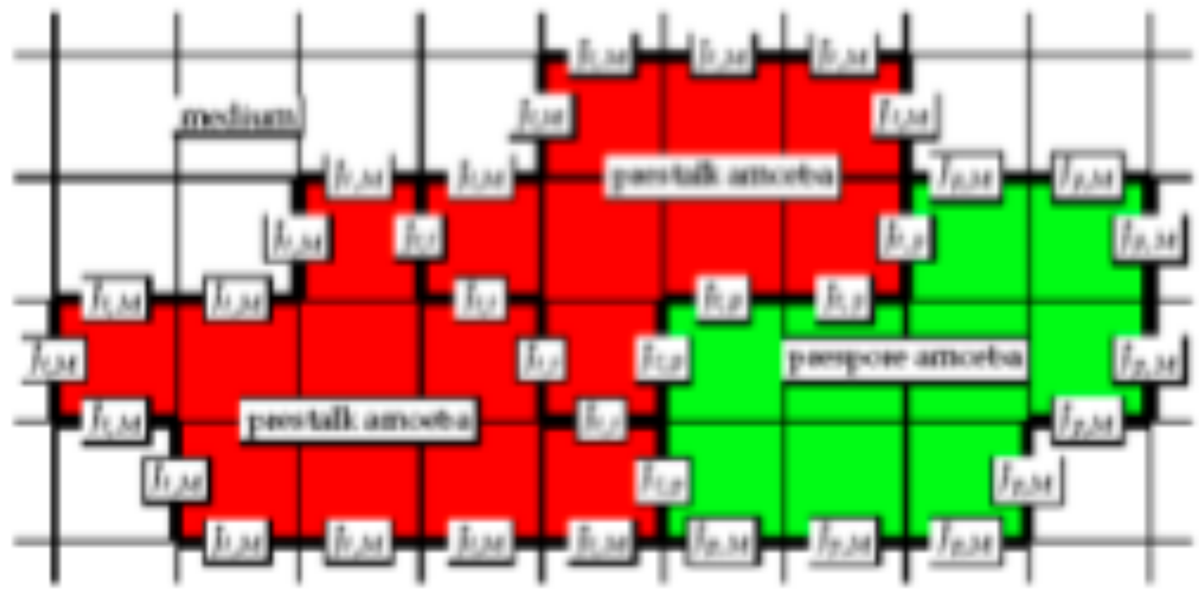
the degrees of freedom of the individual components are increasingly constrained by the interaction:

free living amoebae... slug... immovable sporing pod

nb State space and Phase Space (or: Diagram) are different concepts, but often used interchangeably to describe a State Space… see slide 18

From Pattern Formation to Morphogenesis
Multicellular Coordination in *Dictyostelium Discoideum*
A.F.M. Marée (2000). PhD Thesis, UU.



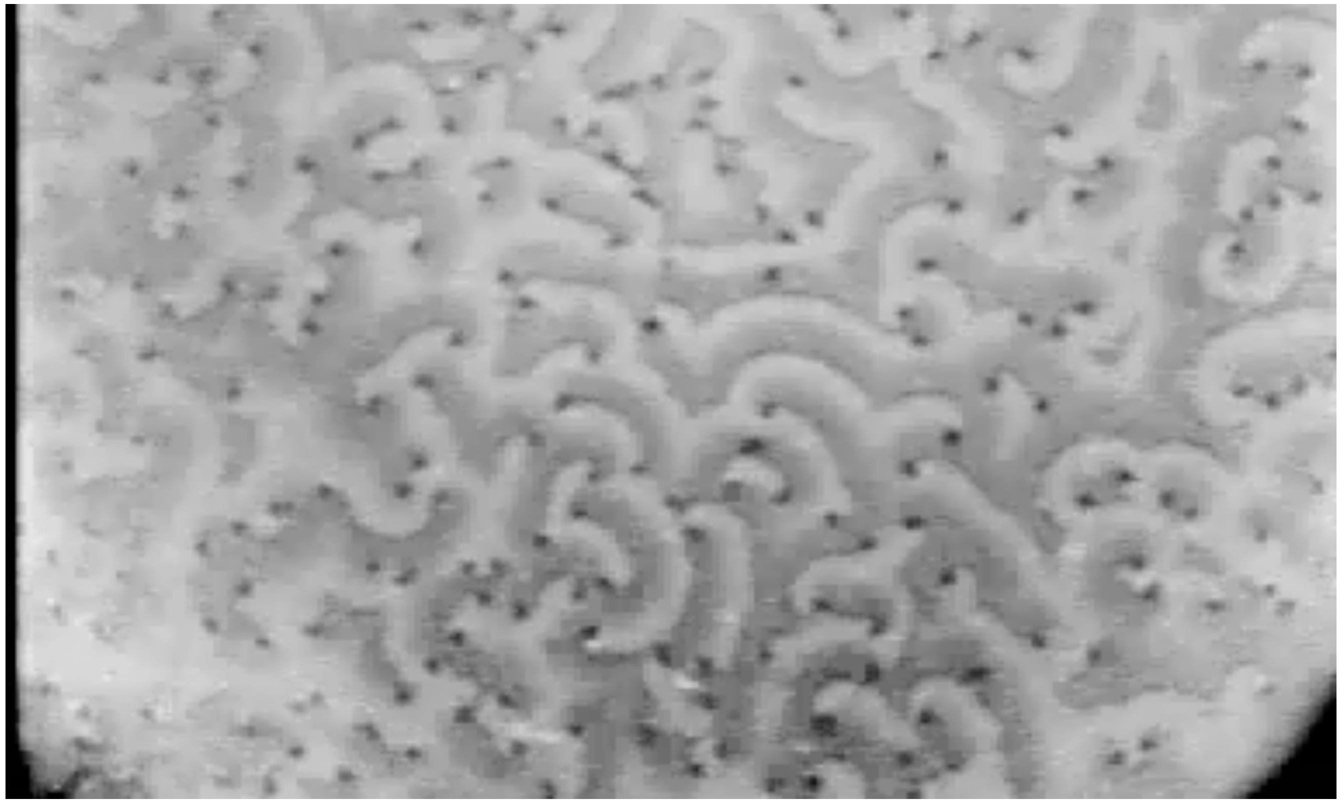


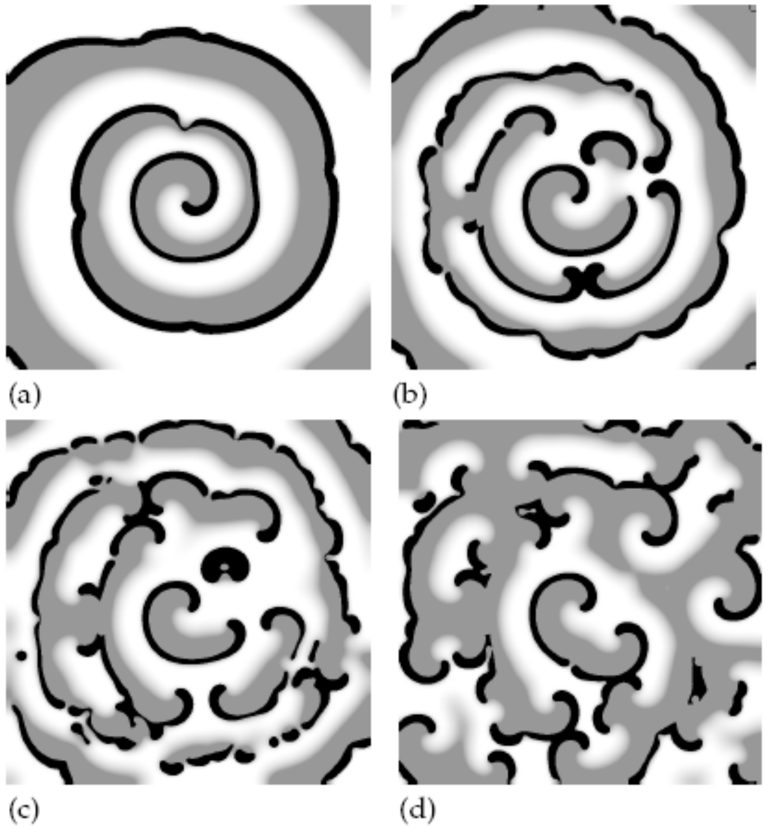
Two-Scale Cellular Automata with Differential Adhesion

$$H_\sigma = \sum_{\text{all } \sigma,\sigma' \text{ neighbours}} \frac{J_{\tau_\sigma,\tau_{\sigma'}}}{2} + \sum_{\text{all } \sigma,\text{ medium neighbours}} J_{\tau_\sigma,\tau_{\text{medium}}} + \lambda(v_\sigma - V)^2, \quad (1.1)$$

Mathematical model of Dictyostelium

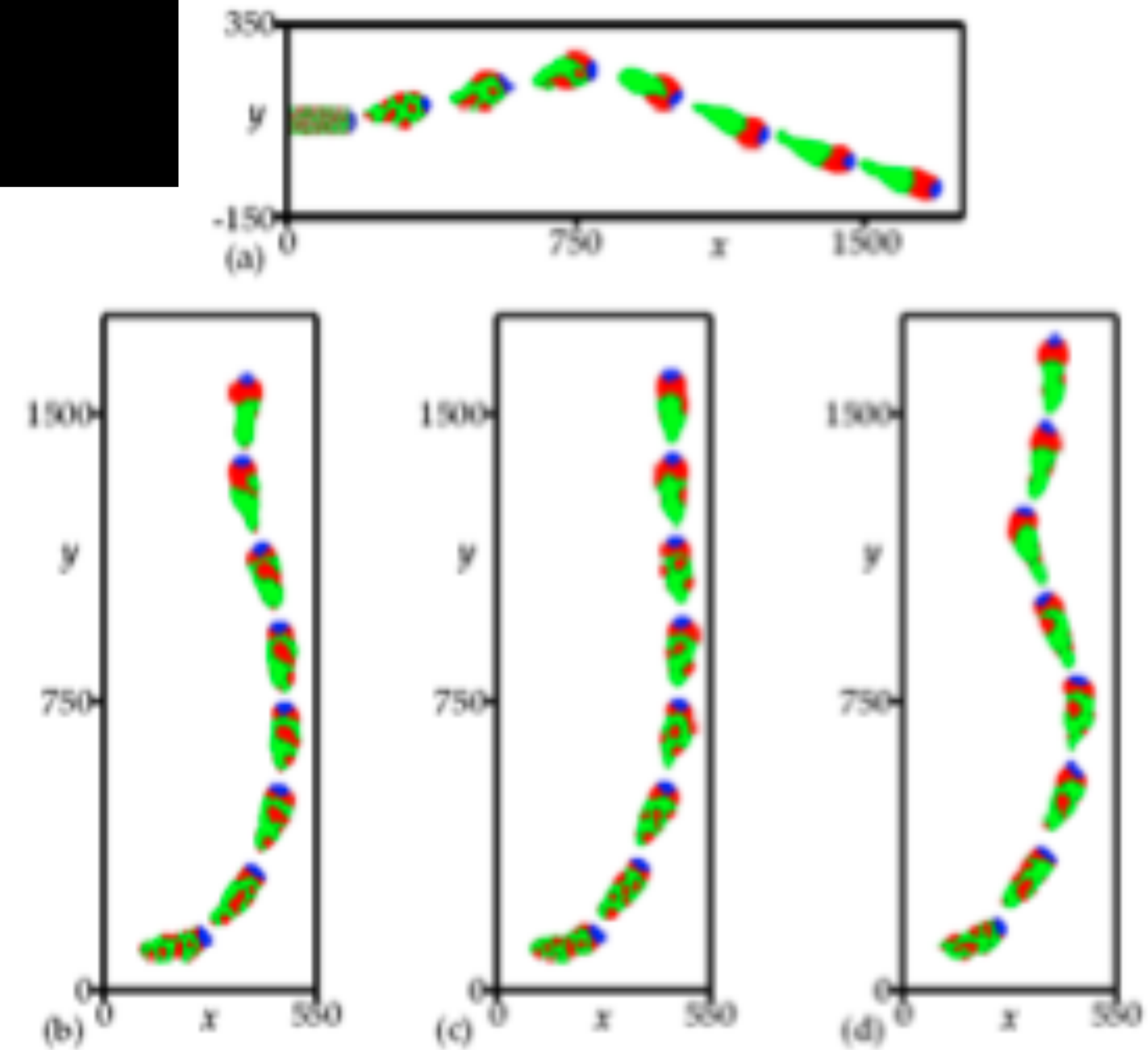
Spiral Breakup in Excitable Tissue due to Lateral Instability
Marée, A. F. M., & Panlov, A.V. (1997). *Physical Review Letters, 78*,1819-1822.





$$\frac{\partial e}{\partial t} = \Delta e - f(e) - g,$$
$$\frac{\partial g}{\partial t} = D_g \Delta g + \varepsilon(e,g)(ke - g),$$

Mathematical model of Dictyostelium



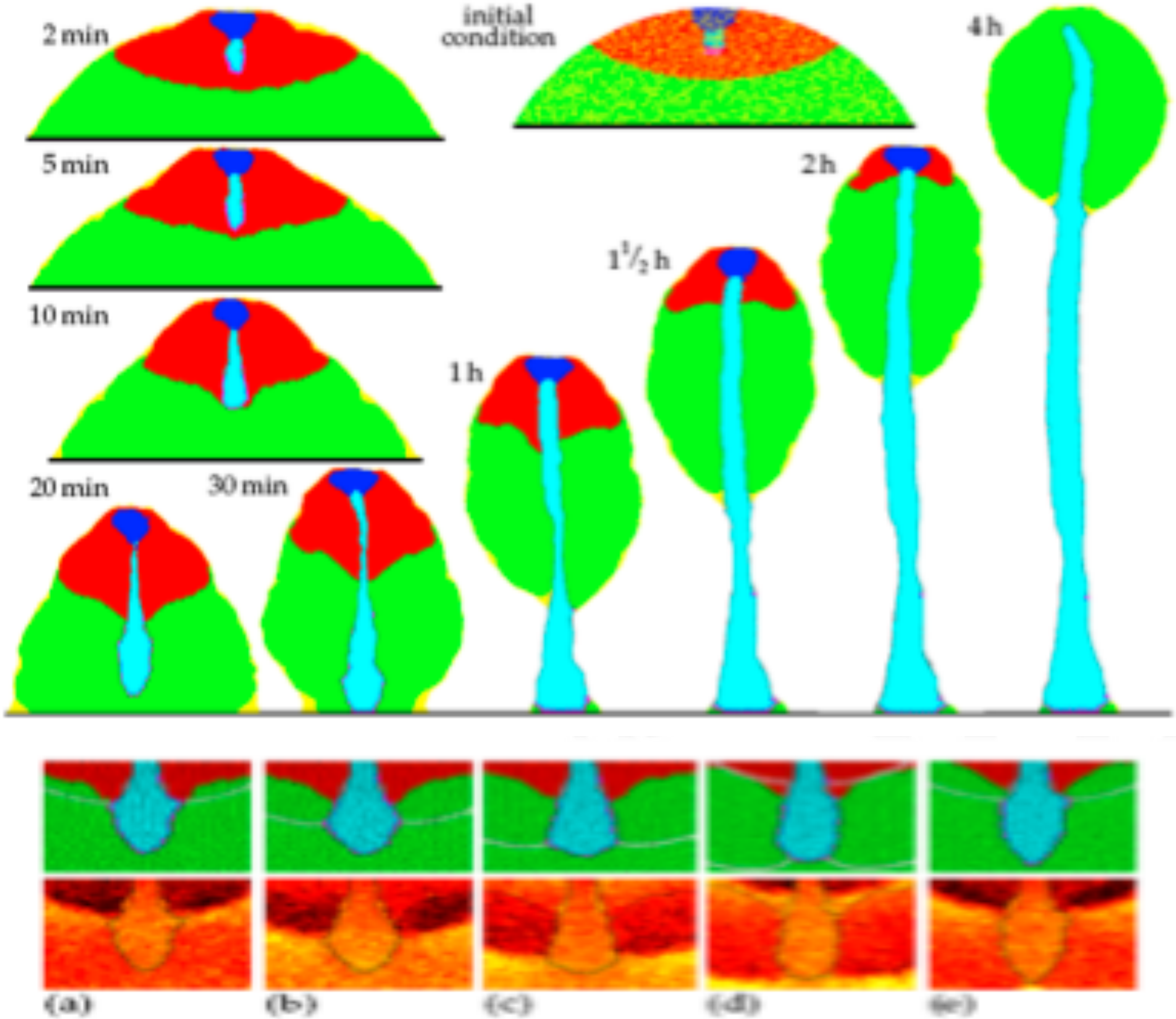
$$H_\sigma = \sum \frac{J_{\text{cell,cell}}}{2} + \sum J_{\text{cell,medium}} + \lambda(v - V)^2,$$

$$\left.\begin{aligned} \frac{\partial c}{\partial t} &= D_c \Delta c - f(c) - r, \\ \frac{\partial r}{\partial t} &= \varepsilon(c)(kc - r), \end{aligned}\right\} \text{ inside the amoebae}$$

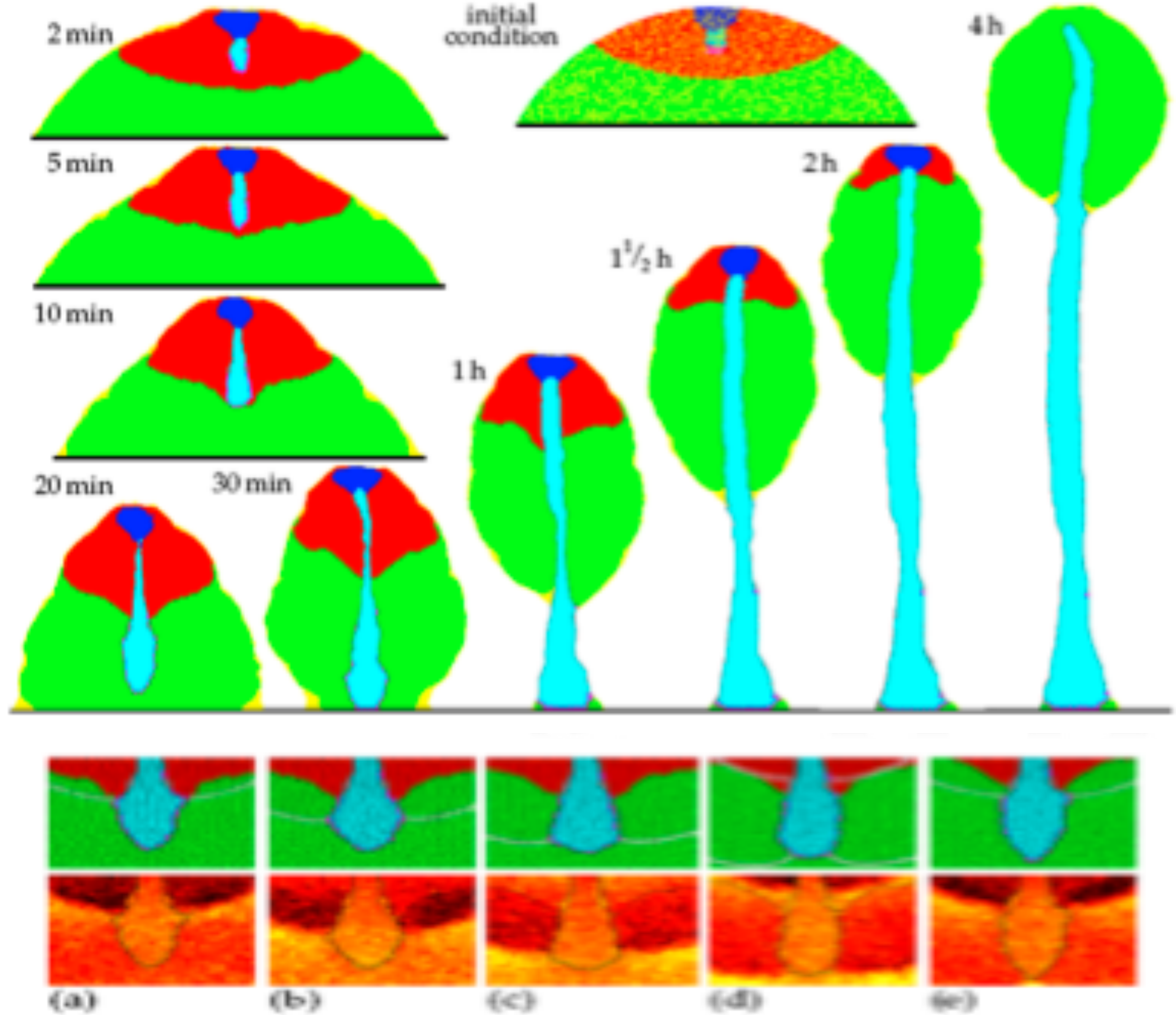
$$\left.\frac{\partial c}{\partial t} = D_c \Delta c - d_c(c - c_0),\right\} \text{ outside the amoebae}$$

$$\Delta H' = \Delta H - \mu(c_{\text{automaton}} - c_{\text{neighbour}}),$$

Mathematical model of Dictyostelium

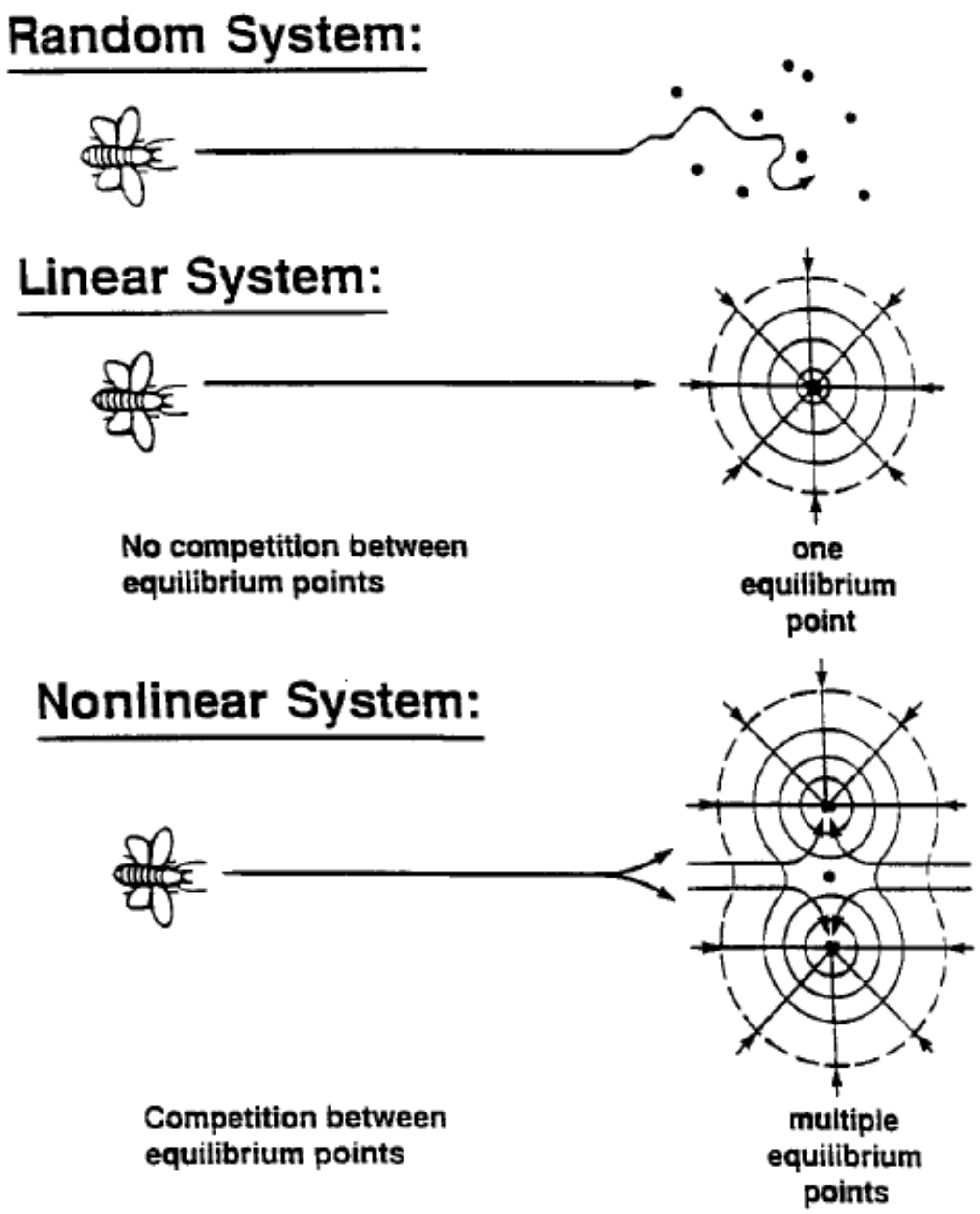


Mathematical model of Dictyostelium

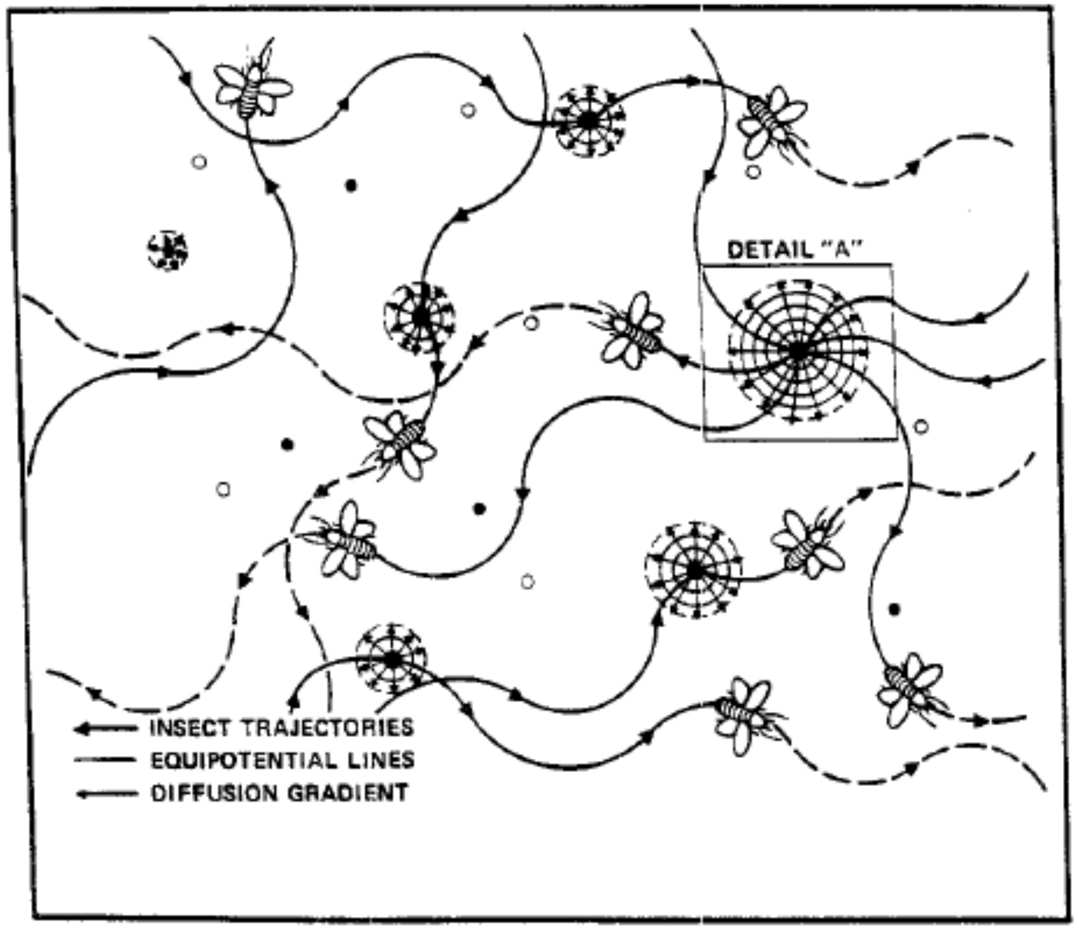


Mathematical model of Dictyostelium

Termite cathedrals: Complex structures from simple rules



Termite cathedrals: Complex structures from simple rules



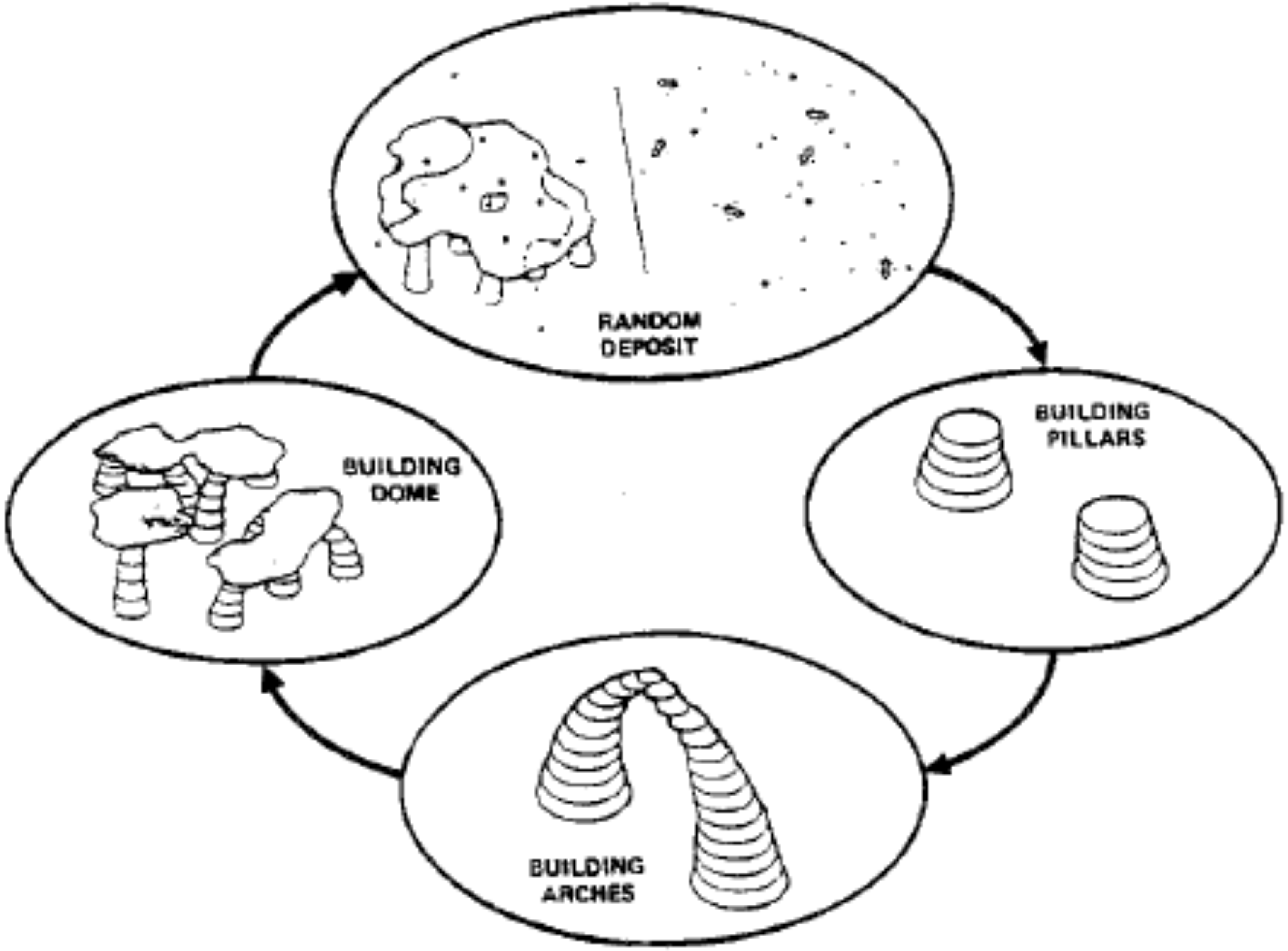


Fig. 14. Circular ring of building phases: each phase is dominated by a

Termite cathedrals: Complex structures from simple rules

Can be “explained” by (local) laws of thermodynamics... termite is a particle in a gradient field...

Dissipative systems: Systems that extract energy from the environment to maintain their internal structure, their internal complexity

Usually: many simple units interact in simple ways to create complex patterns at the global, macro level...

But termites are more complex than classical particles!

Two types of mathematical formalism:

Random events / processes
Linear
Efficient causes

component dominant dynamics

The Law of Large Numbers (Bernouiili, 1713) +
The Central Limit Theorem (de Moivre, 1733) +
The Gauss-Markov Theorem (Gauss, 1809) +
Statistics by Intercomparison (Galton, 1875) =
Social Physics *(Quetelet, 1840)*

Collectively known as:
The Classical Ergodic Theorems

Molenaar, P.C.M. (2008). On the implications of the classical ergodic theorems: Analysis of developmental processes has to focus on intra individual variation. *Developmental Psychobiology, 50*, 60-69

Random events / processes
Deterministic events / processes
Linear / Nonlinear
Efficient causes / Circular causality

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
(Prigogine, & Stengers, 1984)

SOC / $\frac{1}{f^{\alpha}}$ noise (Bak, 1987)
(self-organized criticality, interdependent measurements)

Fractal geometry (Mandelbrot, 1988)
(self-similarity, scale free behaviour, infinite variance)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)

Two types of mathematical formalism for two types of systems

component dominant dynamics

Jakob Bernouiiili (1654-1704): [The application of the Law of large numbers in chance theory] *to predict the weather next month or year, predicting the winner of a game which depends partly on psychological and or physical factors or to the investigation of matters which depend on hidden causes, which can interact in a multitude of ways is* ***completely futile!***" Vervaet (2004)

A system is ***ergodic*** *iff:*

The averaged behaviour of an observed variable in a substantial ensemble of individuals (space-average) is expected to be equivalent to the average behaviour of an individual observed over a substantial amount of time (time average)

f.i. Throw 100 dice at once, and then throw 1 die 100 times in a row… The expected value will be similar for both measurements

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
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Aczel's Anti-Foundation Axiom (1988)
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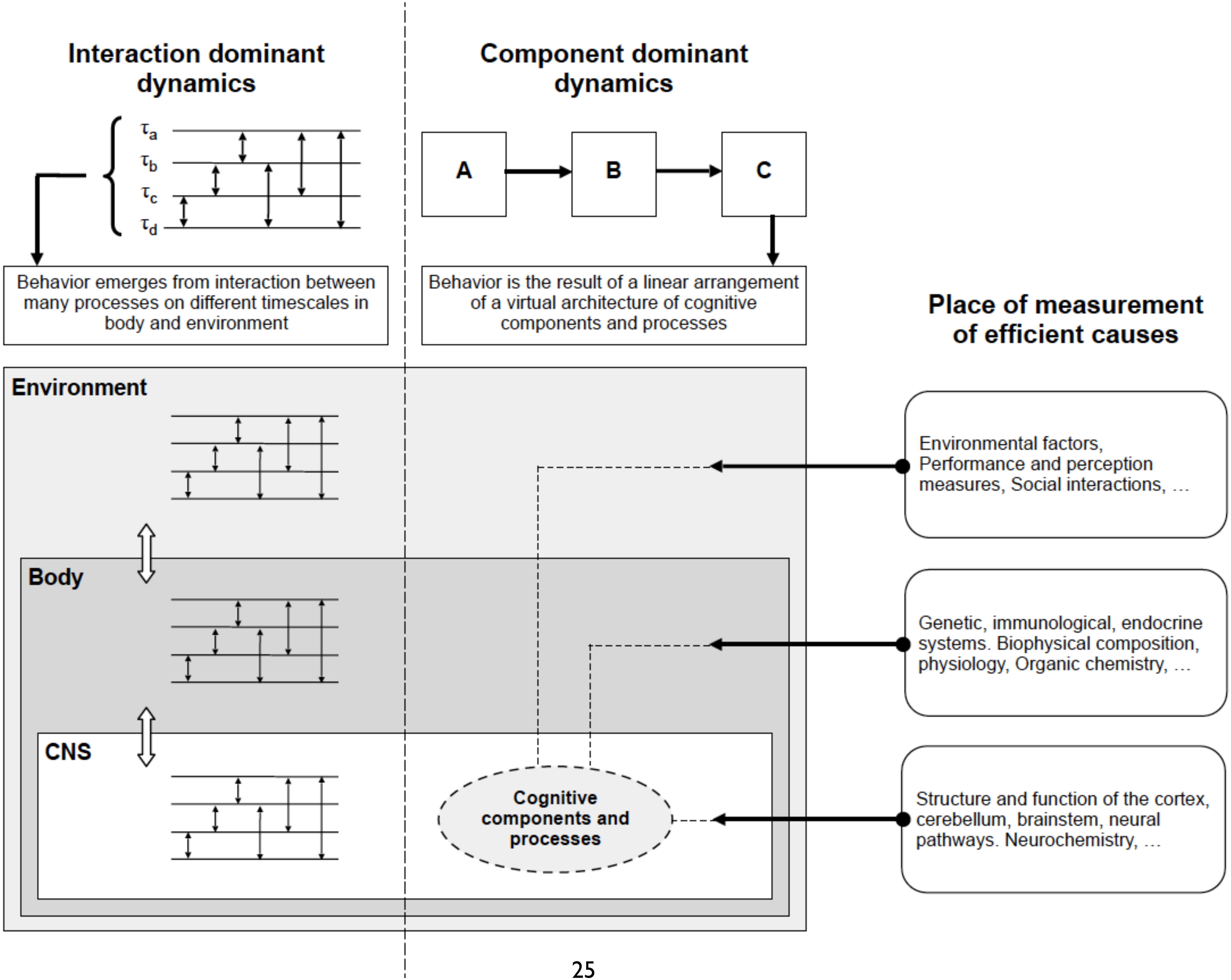


Interaction dominant dynamics
τ_a
τ_b
τ_c
τ_d
Behavior emerges from interaction between many processes on different timescales in body and environment
Component dominant dynamics
A
B
C
Behavior is the result of a linear arrangement of a virtual architecture of cognitive components and processes
Place of measurement of efficient causes
Environment
Body
CNS
Cognitive components and processes
Environmental factors, Performance and perception measures, Social interactions, …
Genetic, immunological, endocrine systems. Biophysical composition, physiology, Organic chemistry, …
Structure and function of the cortex, cerebellum, brainstem, neural pathways. Neurochemistry, …

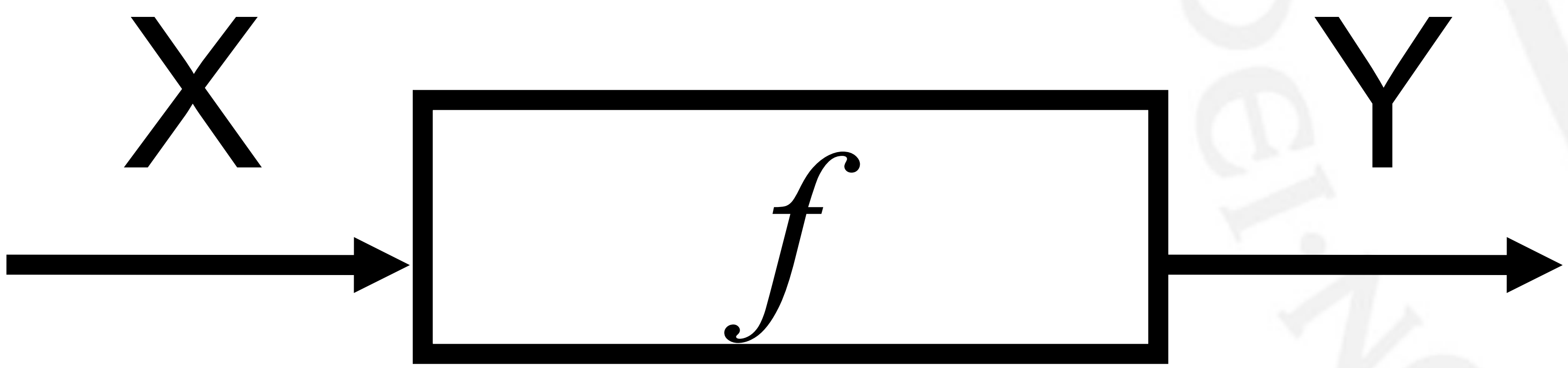
Complexity Methods for Behavioural Science

Day 1: Intro to Complexity Science
Intro Mathematics of Change

The mathematics of change

Traditional: Functional relations

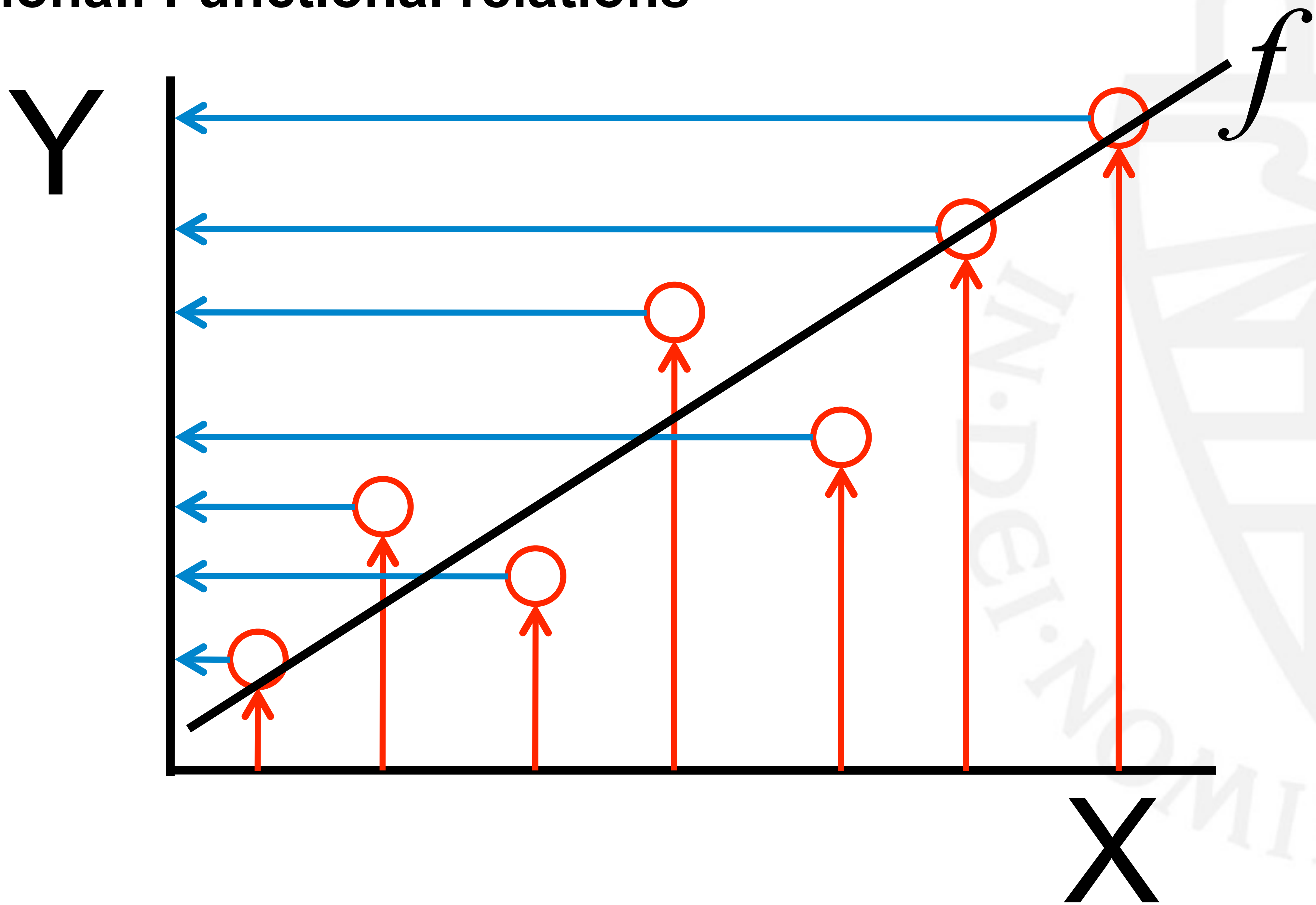
$$Y = f(X)$$



1refs

The mathematics of change

Traditional: Functional relations



The mathematics of change

Complex systems however:

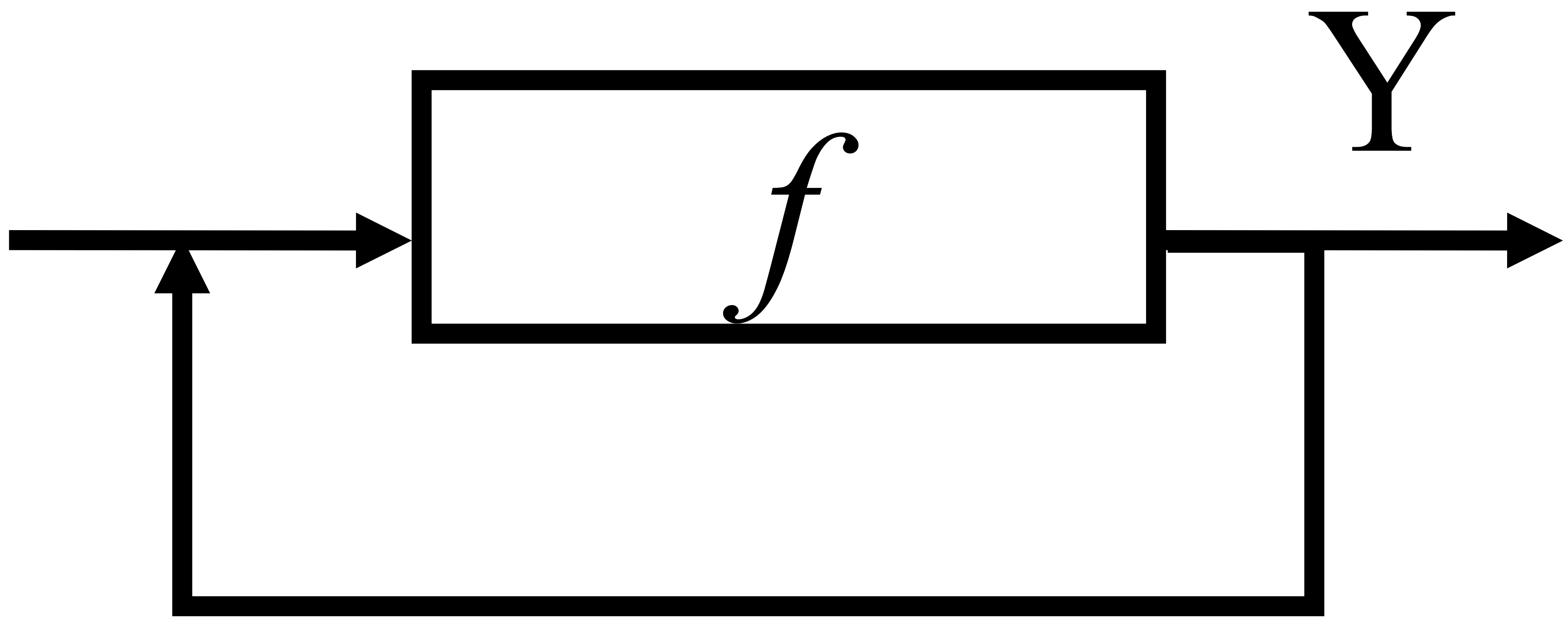
- Consist of feedback loops
- Are recurrent / recursive
- Have history
- Are characterised by multiplicative interactions between components

[1]refs

The mathematics of change

Complex systems: Recurrent processes / Feedback

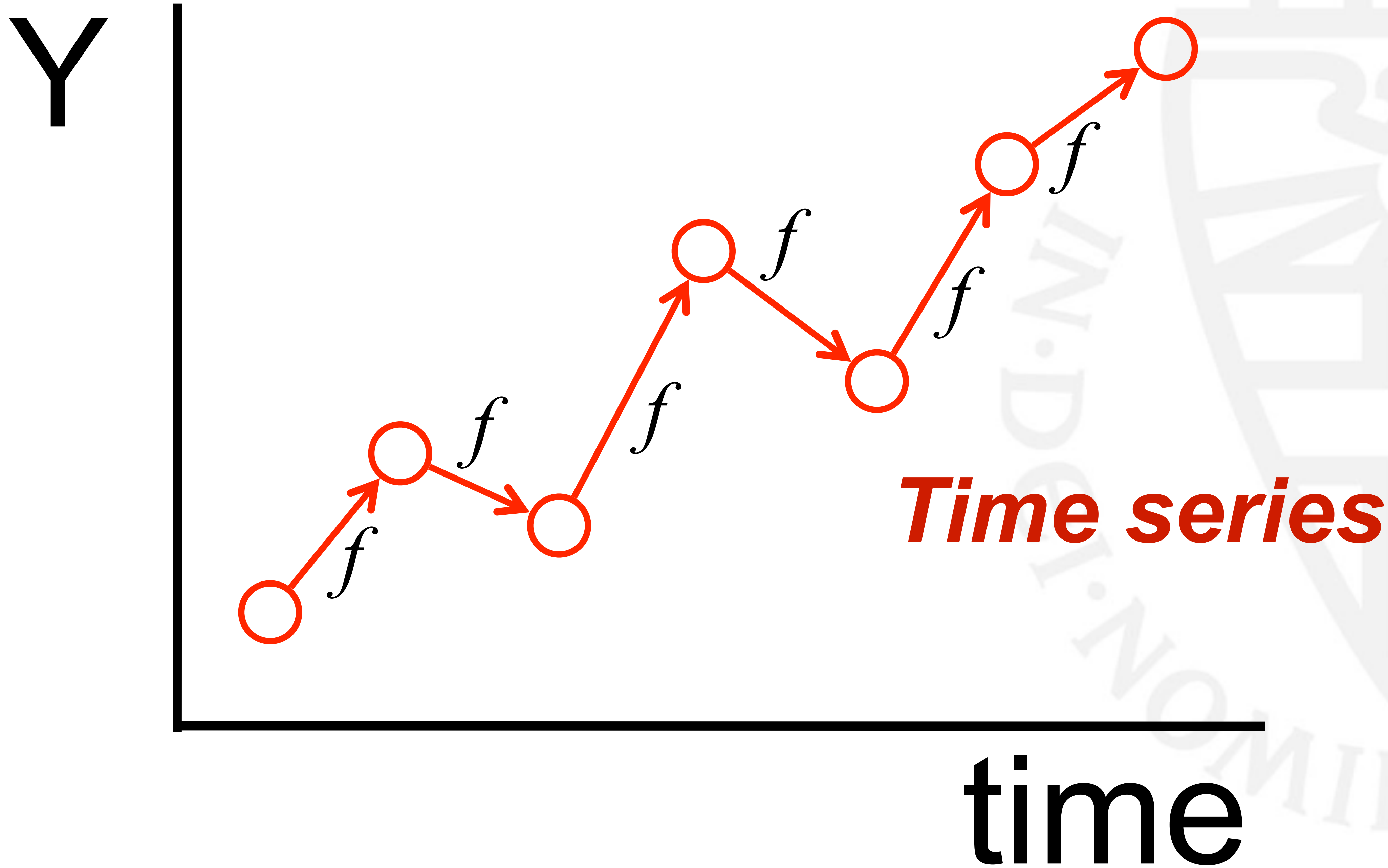
$$\hat{Y} = f(Y)$$



1refs

The mathematics of change

Complex systems: Recurrent processes / Feedback



[1]refs

Two Flavors: Flows & Maps

Dynamical models of psychological processes can be formulated in:

‘Clock’ time

Continuous System
~ Flow ~
(Differen*tial* equation)

‘Metronome’ time

Discrete System
… Map ...
(Differen*ce* equation)

1refs

Parameters & Bifurcations

Example 1:

The Linear Map

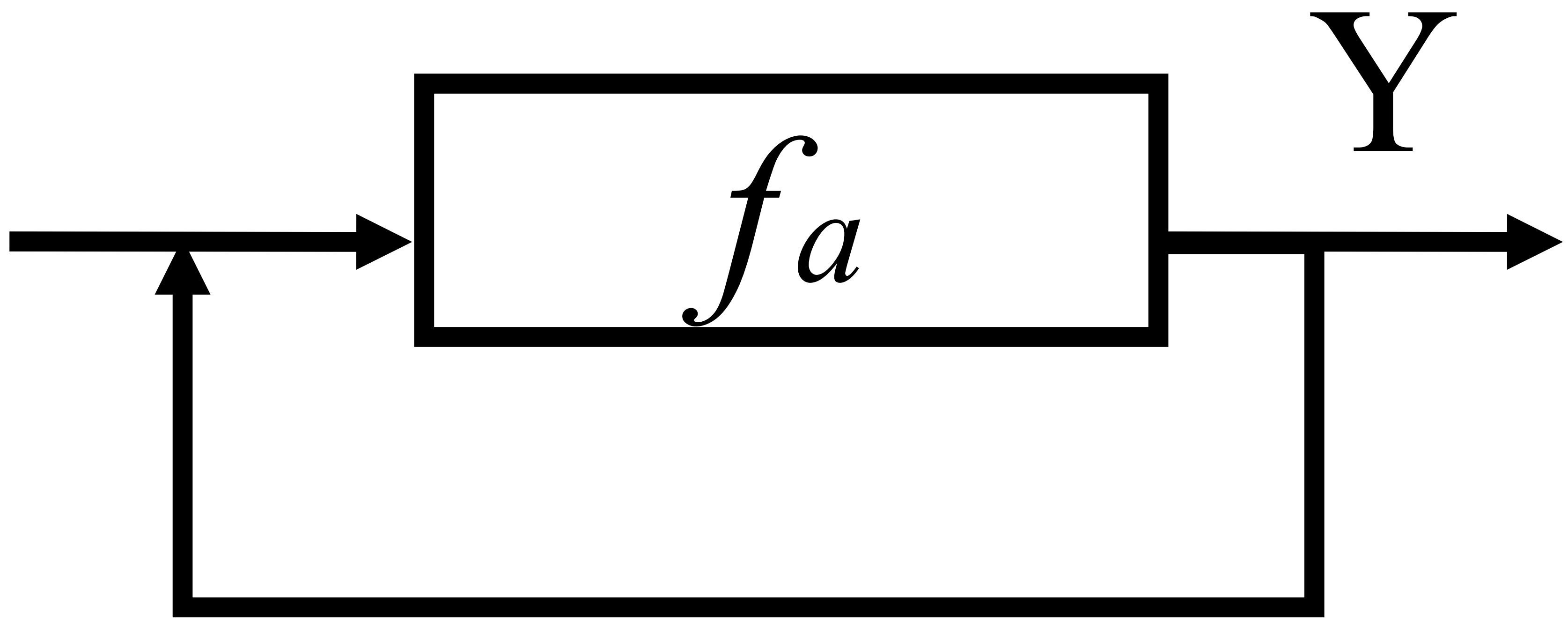
(Linear Growth)

1refs

The linear map

Dynamic Models: Parameter

$$\hat{Y} = f_a(Y)$$



The Linear Map …

The (rate of) change of the state of a system is proportional to its current state:

$$Y_i+1 = a \cdot Y_i$$

...Iteration...

[1]refs

The Linear Map

$\boxed{\text{Initial value: } Y_0}$

$\boxed{Y_1 = f(Y_0)}$

$\boxed{Y_2 = f(Y_1)}$

$\boxed{Y_3 = f(Y_2)}$

Iteration in general just means applying the function over and over again starting with an initial value

and subsequently to the result of the previous step

[1] refs

The Linear Map

$$\boxed{Y_{i+1} = f(Y_i)}$$

i = 0: $Y_0 \rightarrow Y_1 = f(Y_0)$

i = 1: $Y_1 \rightarrow Y_2 = f(Y_1) = f(f(Y_0)) = f^2(Y_0)$

i = 2: $Y_2 \rightarrow Y_3 = f(Y_2) = \ldots = f^3(Y_0)$

$\vdots \qquad \vdots$

[1]refs

i = n: $Y_n \rightarrow Y_{n+1} = f(Y_n) = \ldots = f^n(Y_0)$

Linear Map: Iteration with a parameter

$$\boxed{Y_{i+1} = a \cdot Y_i}$$

i = 0: $Y_0 \rightarrow Y_1 = a \cdot Y_0$

i = 1: $Y_1 \rightarrow Y_2 = a \cdot Y_1 = a \cdot a \cdot Y_0 = a^2 \cdot Y_0$

i = 2: $Y_2 \rightarrow Y_3 = a \cdot Y_2 = \ldots = a^3 \cdot Y_0$

$\vdots \qquad \vdots$

i = n: $Y_n \rightarrow Y_{n+1} = a \cdot Y_n = \ldots = a^{n+1} \cdot Y_0$

[1]refs

Linear Map: Iteration with a Parameter

$$Y_{i+1} = a \cdot Y_i$$

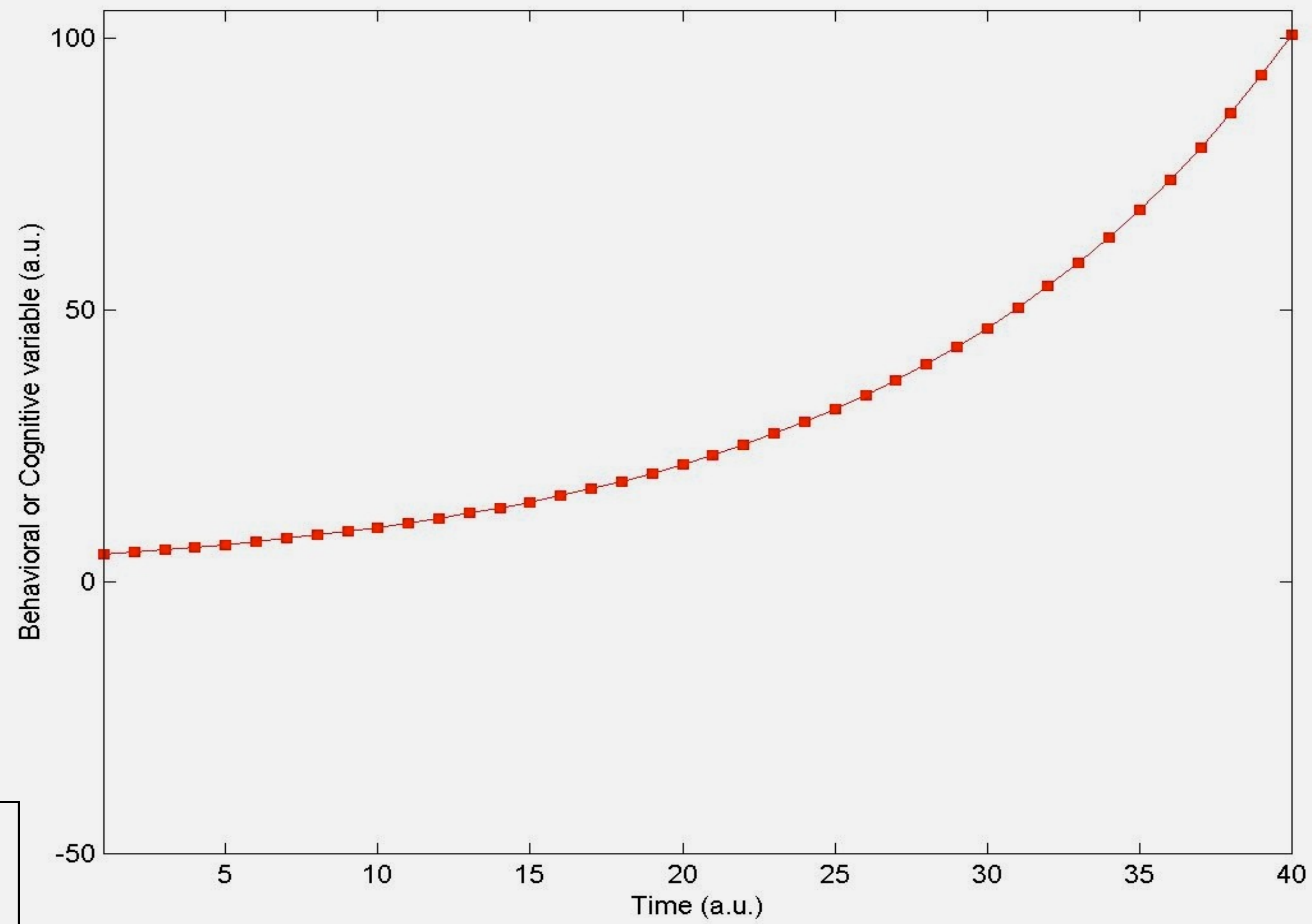
$0 < a < 1$	$-1 < a < 0$
$a > 1$	$a < -1$
$a = 1$	$a = -1$

Y_0 nonspecific

[1]refs

Linear Map: Iteration with a Parameter

a = 1.08
Y0 = 5

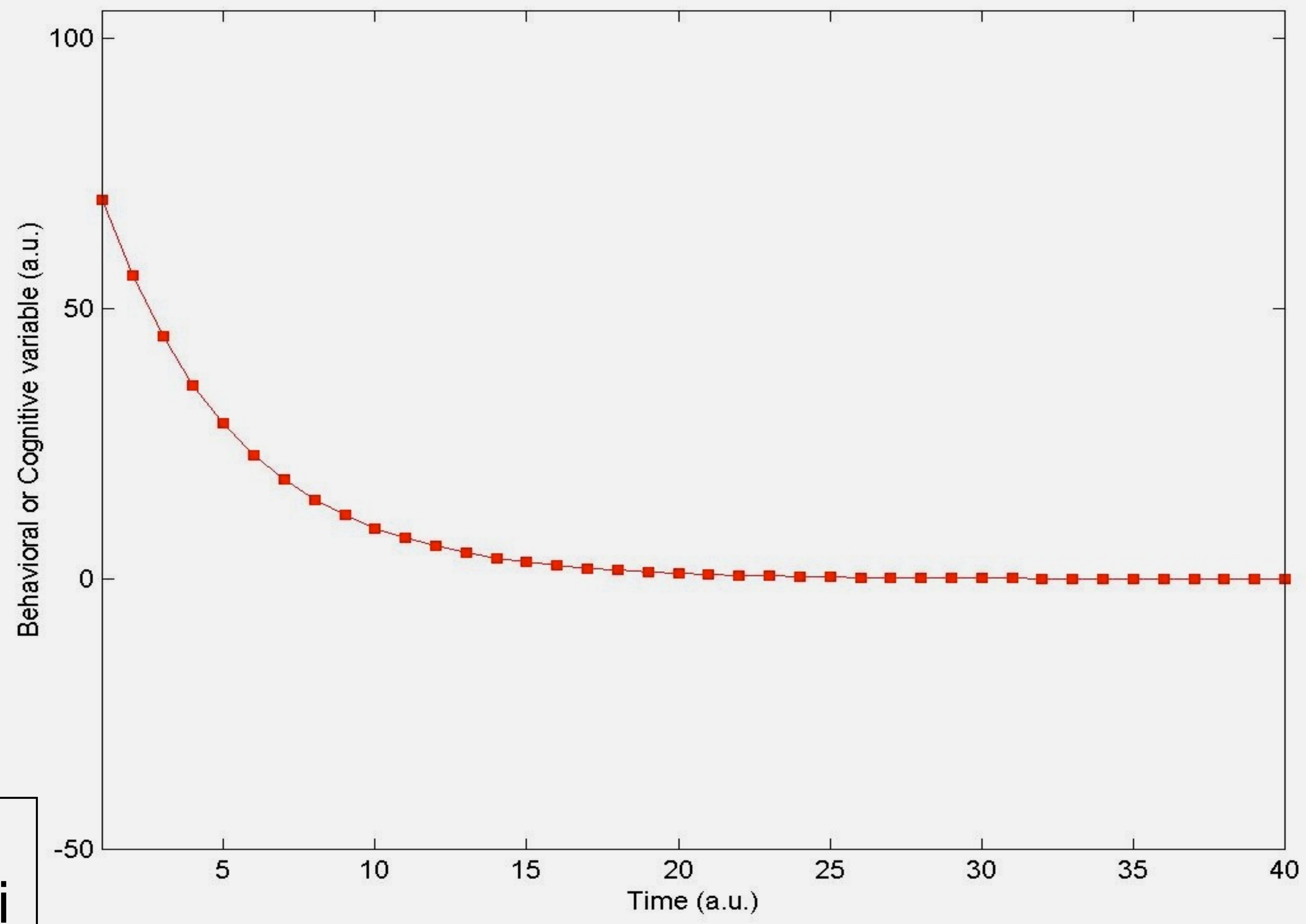


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

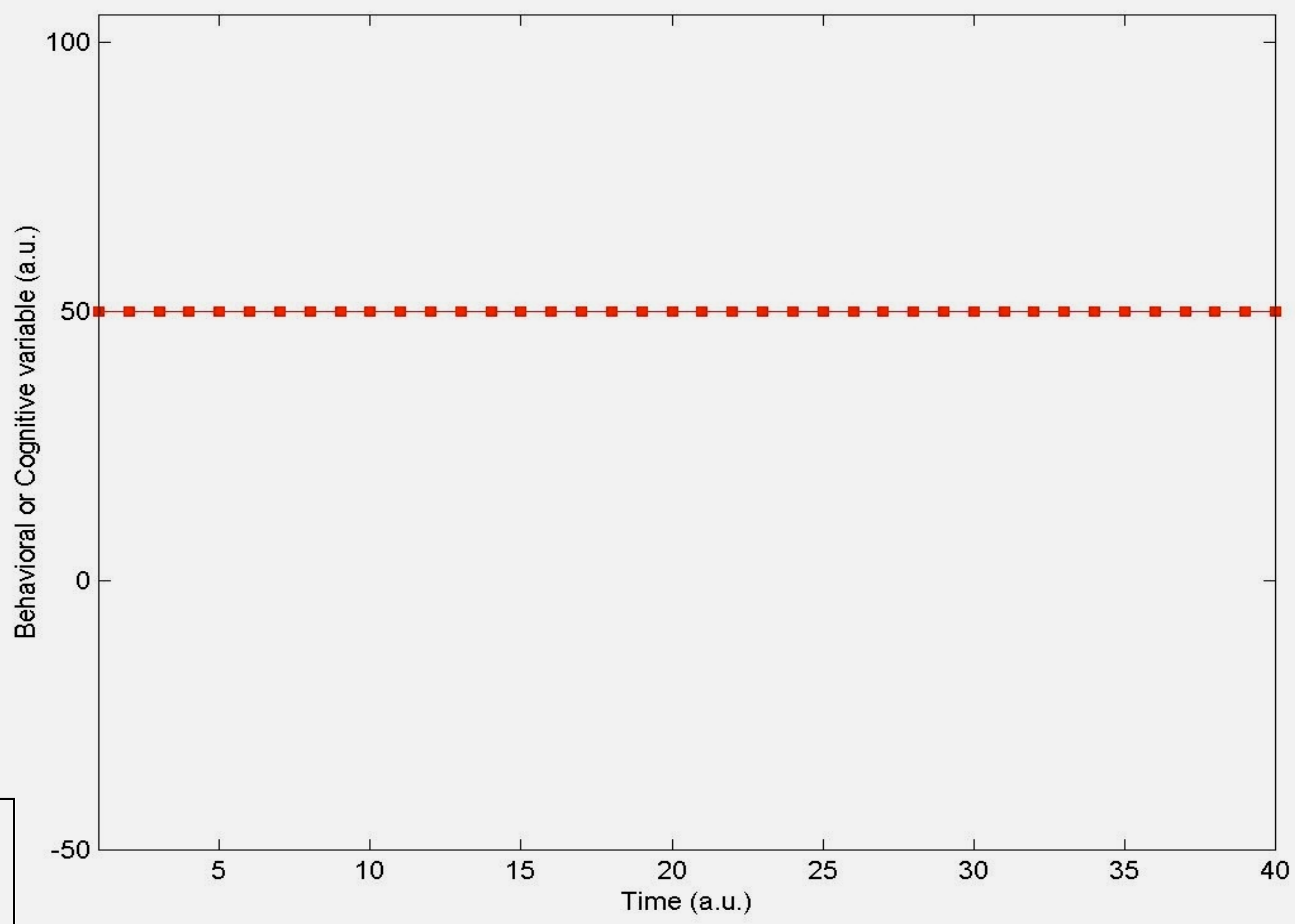
a = 0.8
Y0 = 70



$$Y_{i+1} = a \cdot Y_i$$

Linear Map: Iteration with a Parameter

a = 1.00
Y0 = 50

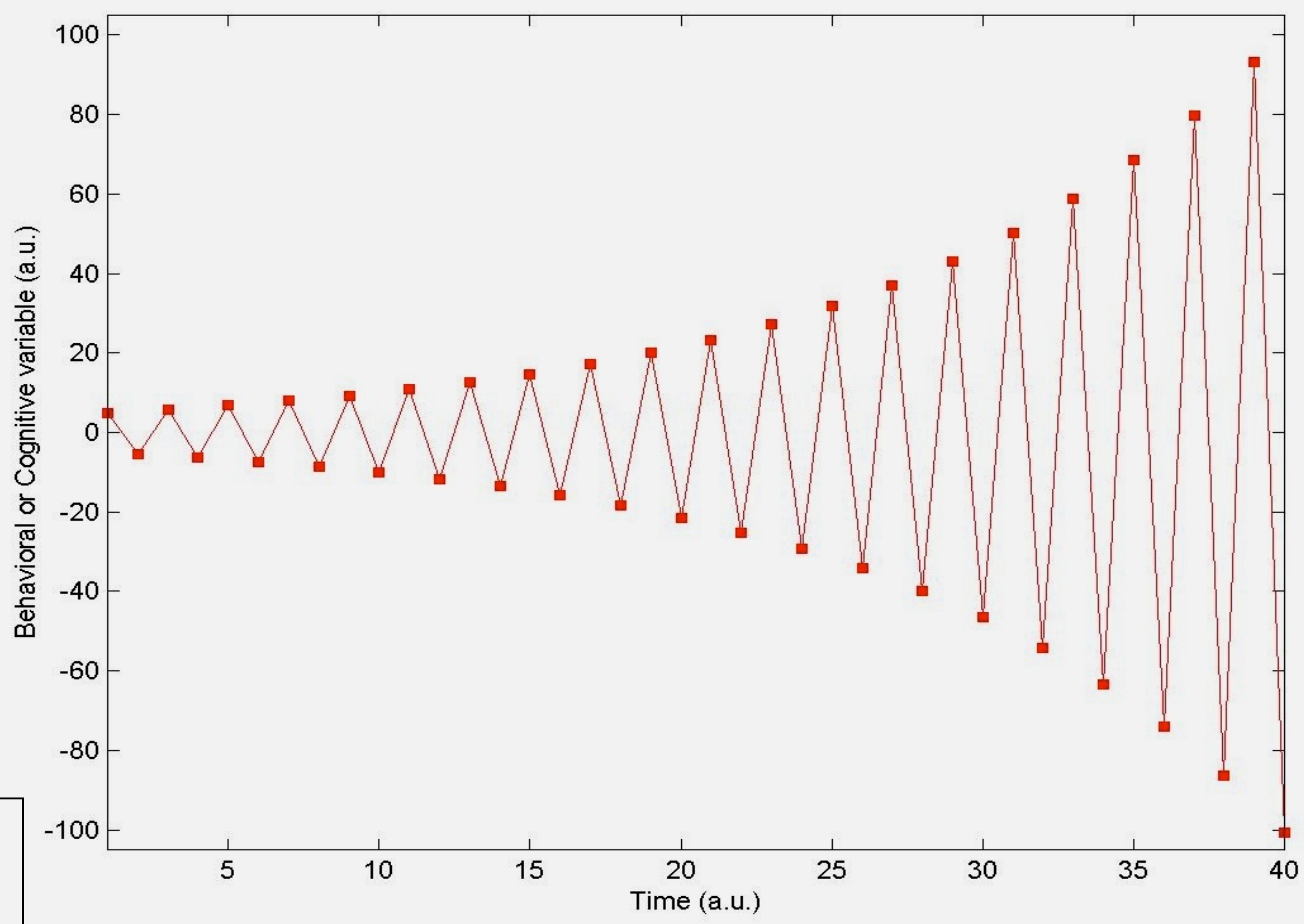


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.08
Y0 = 5

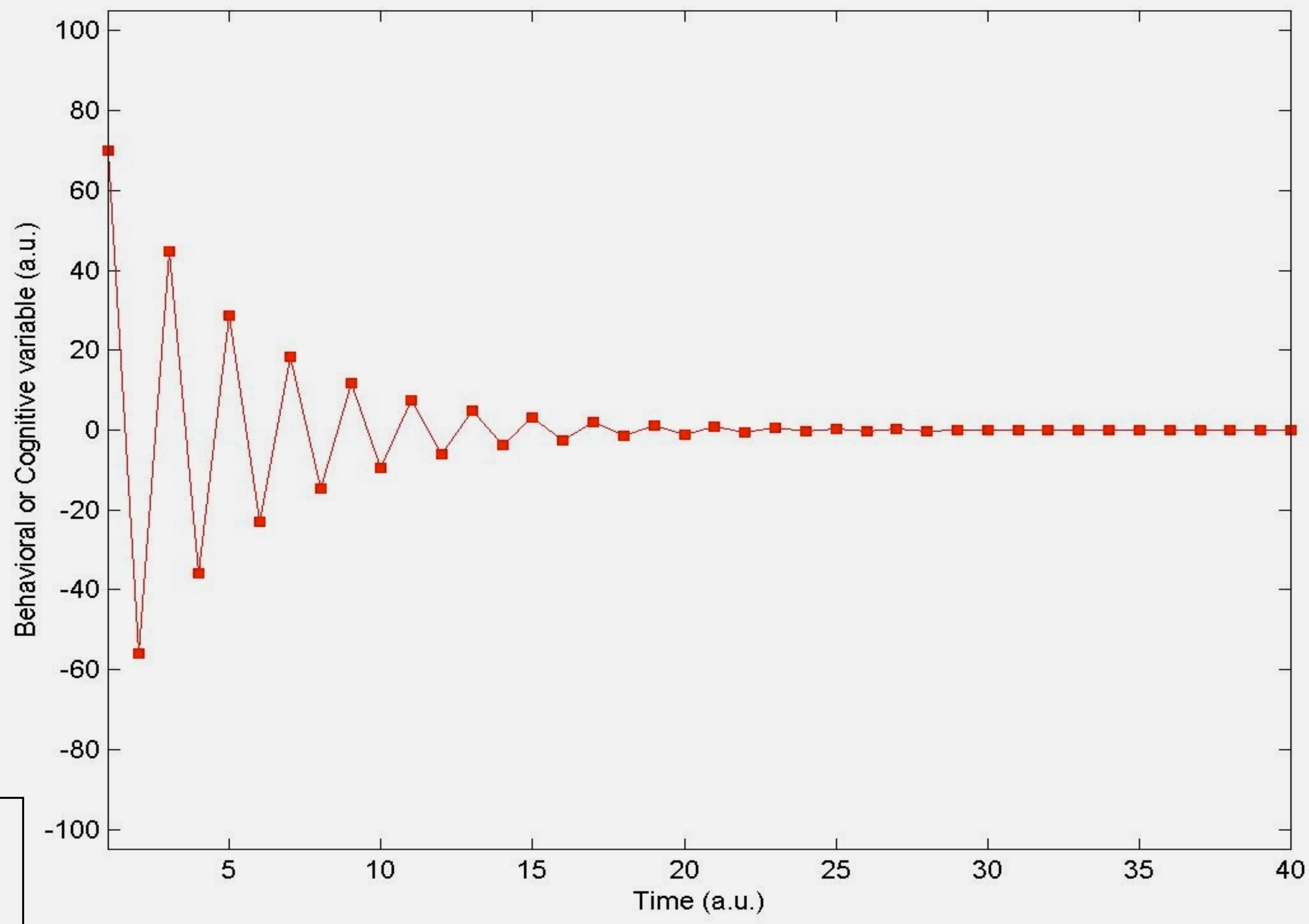


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -0.8
Y0 = 70

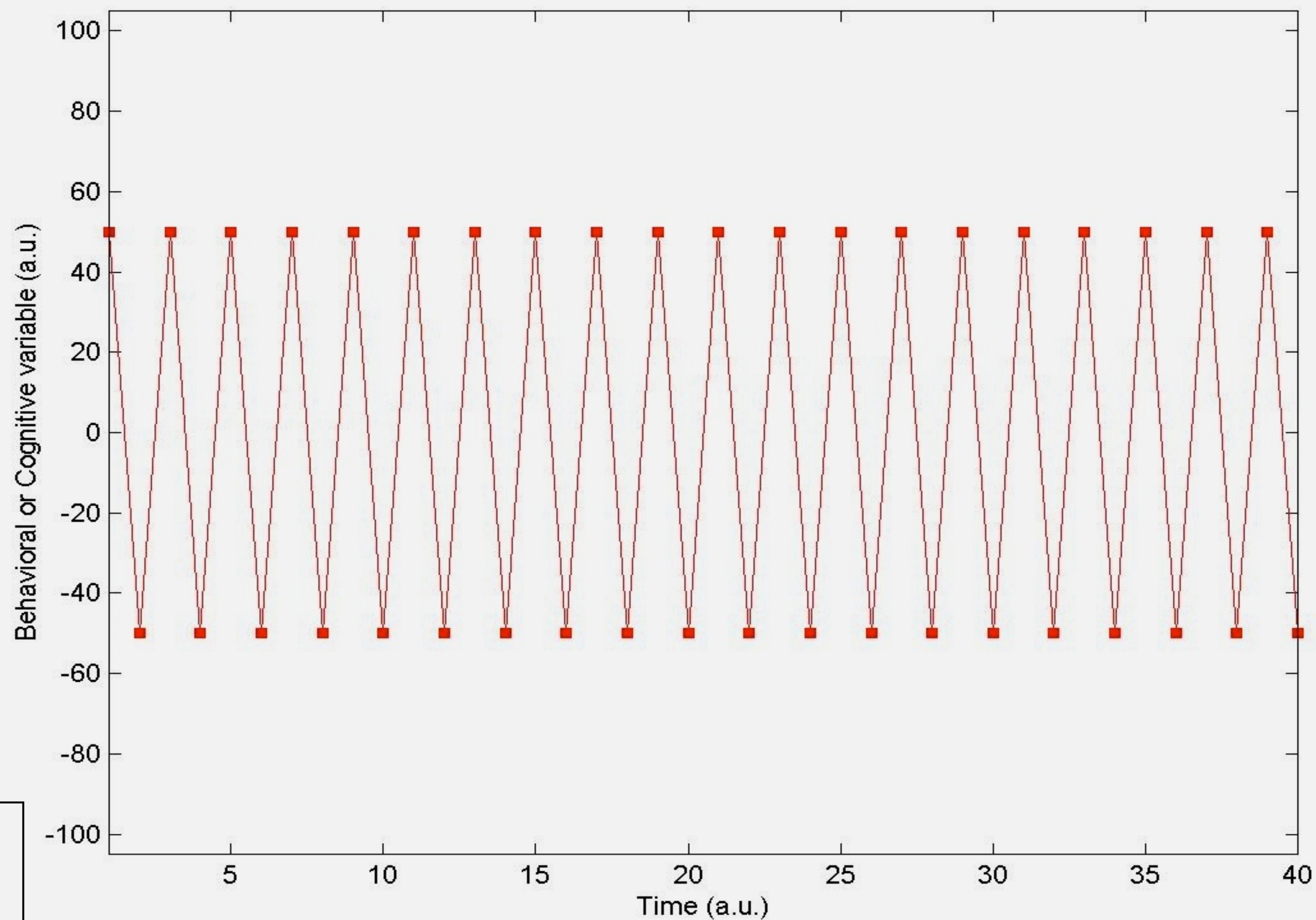


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.00
Y0 = 50



$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

Some interesting differences compared to a linear model:

- Change of behaviour over iterations
  - *Simple model vs. "time" or "occasion" as a predictor*
- Qualitatively different behaviour
  - *One model produces at least four different types of behaviour*
  - *Not by adding predictors (components), by changing one parameter*

[1]refs

Parameters & Bifurcations

Example 2:

The Logistic Map

(restricted growth)

Logistic Map …

$$\boxed{L_{i+1} = r L_i (1 - L_i)}$$

- Simplest nontrivial model often used as an introduction to DST and Chaos theory.
- Well-known model in ecology, physics, economics and social sciences.
- ‘Styled’ version of Van Geert’s model for language growth. (*Next meeting*)

[1]refs

Logistic Map: Iteration

$$\boxed{L_{i+1} = r L_i (1 - L_i)}$$

i = 0: $L_0 \rightarrow L_1 = r L_0 (1 - L_0)$

i = 1: $L_1 \rightarrow L_2 = r L_1 (1 - L_1)$

$$= r\, rL_0(1-L_0)\,(1 - rL_0(1-L_0))$$

$$= -r^3L_0^4 + 2r^3L_0^3 - r^2(1+r)L_0^2 + r^2L_0$$

[1]refs

Logistic Map: Parameter

$$L_{i+1} = r\,L_i\,(1 - L_i)$$

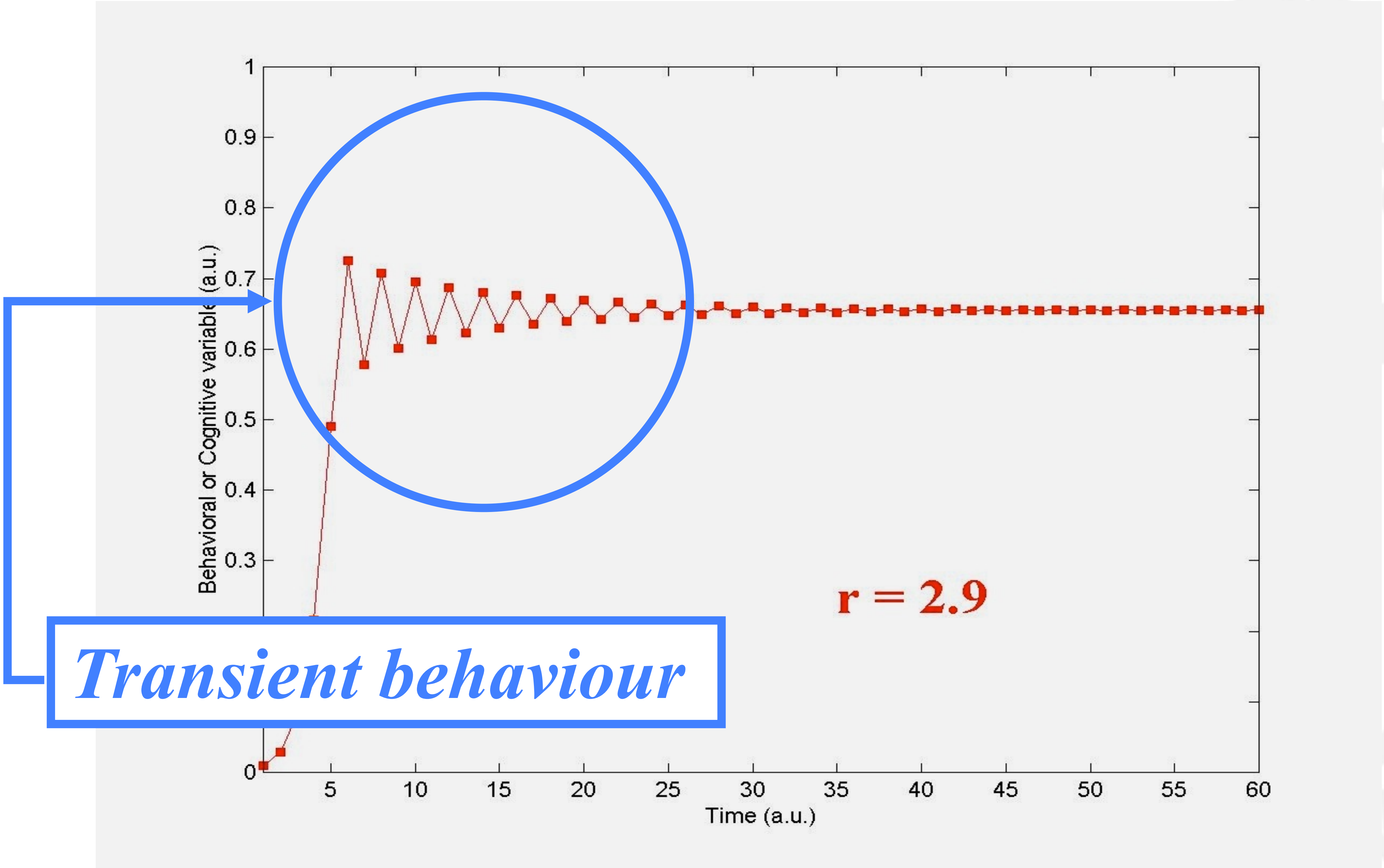
$r = 0.90$	$r = 3.30$
$r = 1.90$	$r = 3.52$
$r = 2.90$	$r = 3.90$

L_0 small

[1]refs

Logistic Map: Graphs

$$L_{i+1} = r\,L_i\,(1 - L_i)$$



[1]refs

An ecology of growth models? Same principle!

Basic Growth Models: Exponential + Restricted Growth

$$Population = rN \times (\frac{K - N}{K})$$

Additional Parameter: Carrying Capacity

$$CognitiveGrowth = L_i (1 + r \times \frac{K - L_i}{K})$$

$$StylizedLogistic = rY_i \times (\frac{1 - Y_i}{1})$$

Bifurcation Diagram

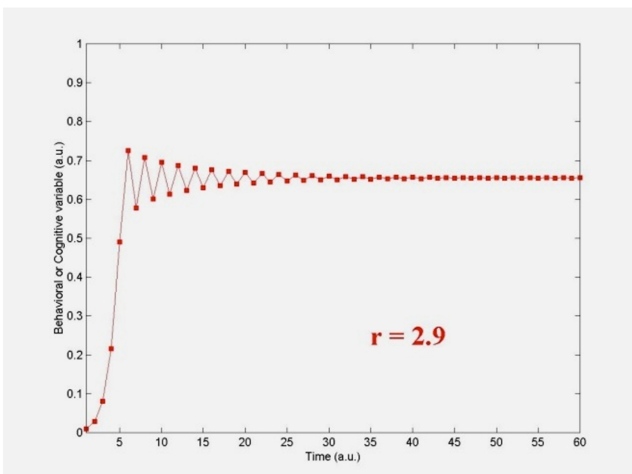
Bifurcation Diagram - Phase Diagram

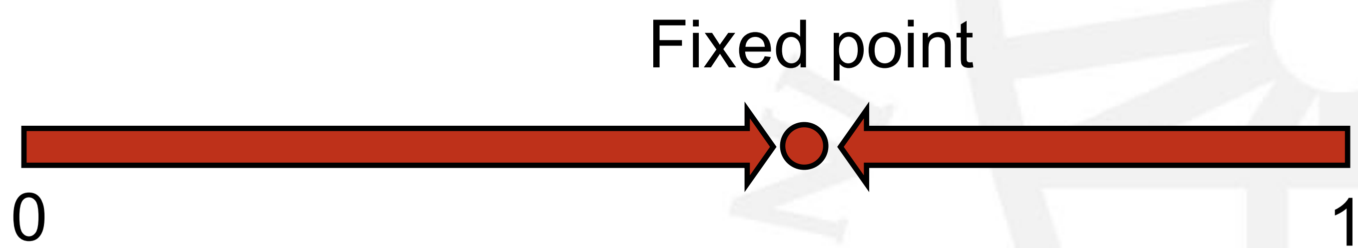
A graphical representation of the possible states a dynamical system can end up in for different values of one or more parameters.

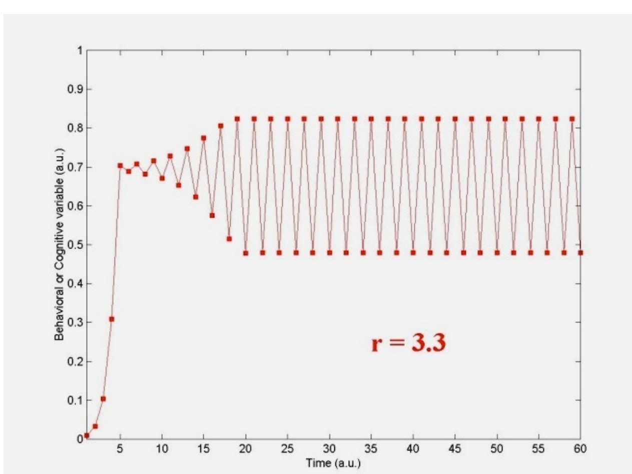
- The parameter is called the ***control parameter***.
- The end states are called ***attractors***.
- The change from one attractor (or set) to another is called a ***bifurcation***.

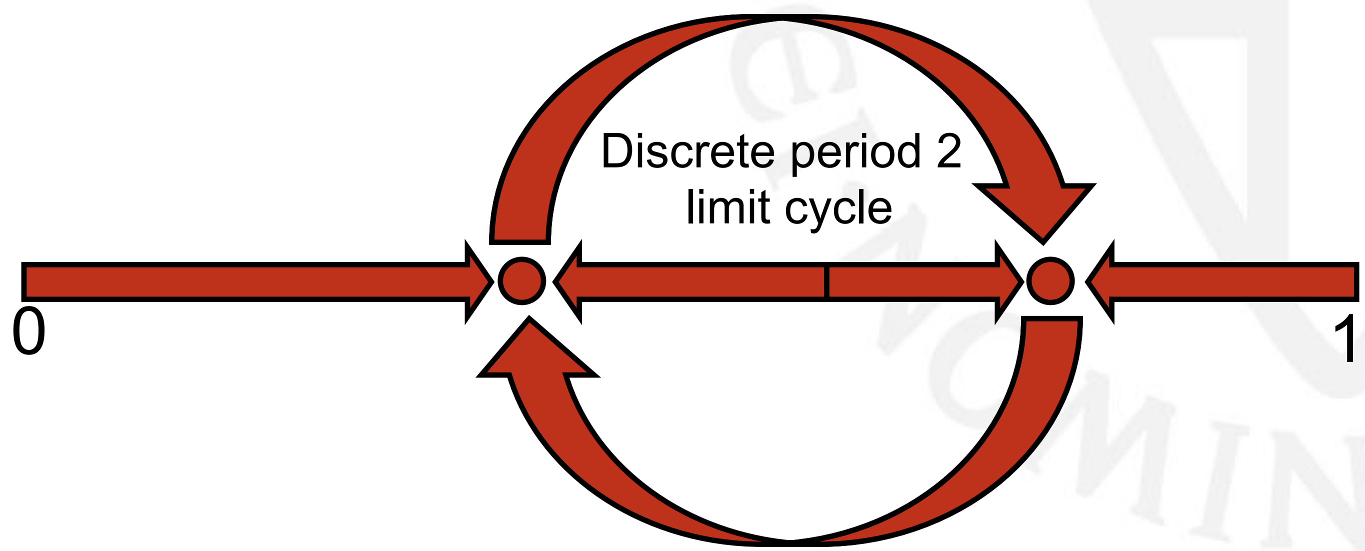
End states are attractors in state space: Attractor types

State Space is an abstract space used to represent the behaviour of a system. Its dimensions are the variables of the system. Thus a point in the phase space defines a potential state of the system. The points actually achieved by a system depend on its iterative function and initial condition (starting point).









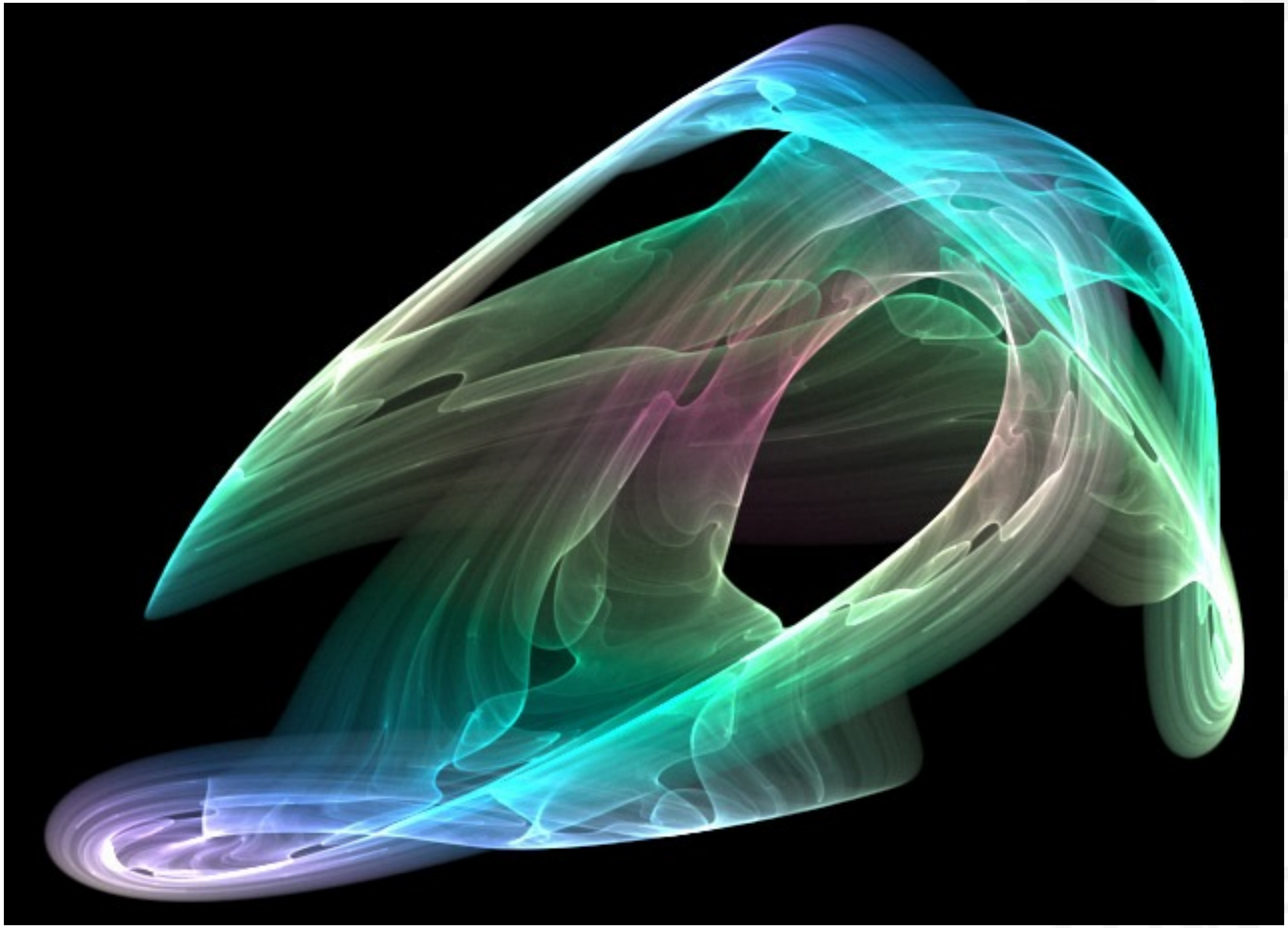
State space, Attractor types

"Saturn"
attractor

Strange attractors
are quasi periodic
and bounded

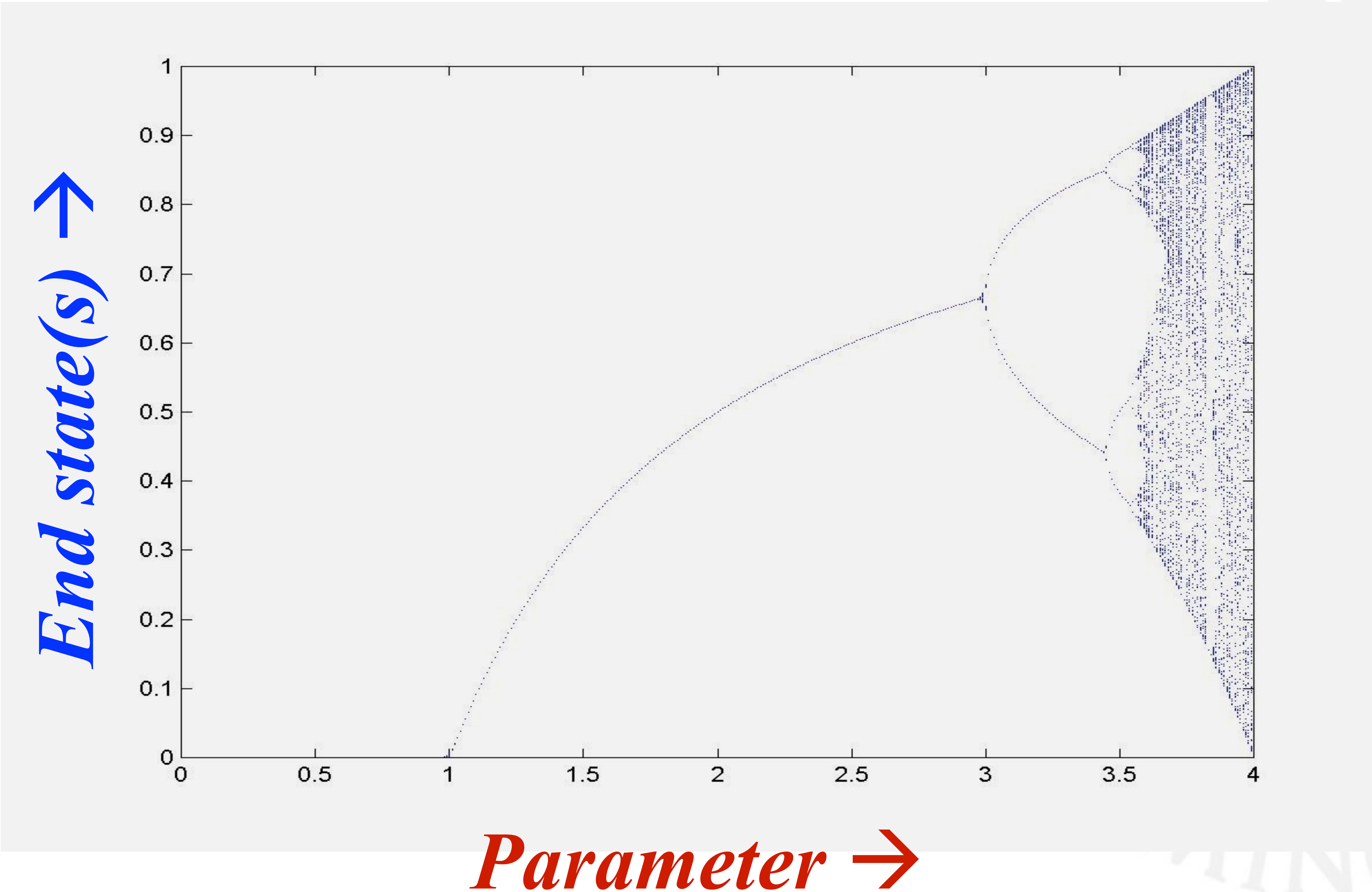
Bottom line:

An attractor means
a limited region
of state space
is visited.
Not all DF actually
available
to the system
are used.



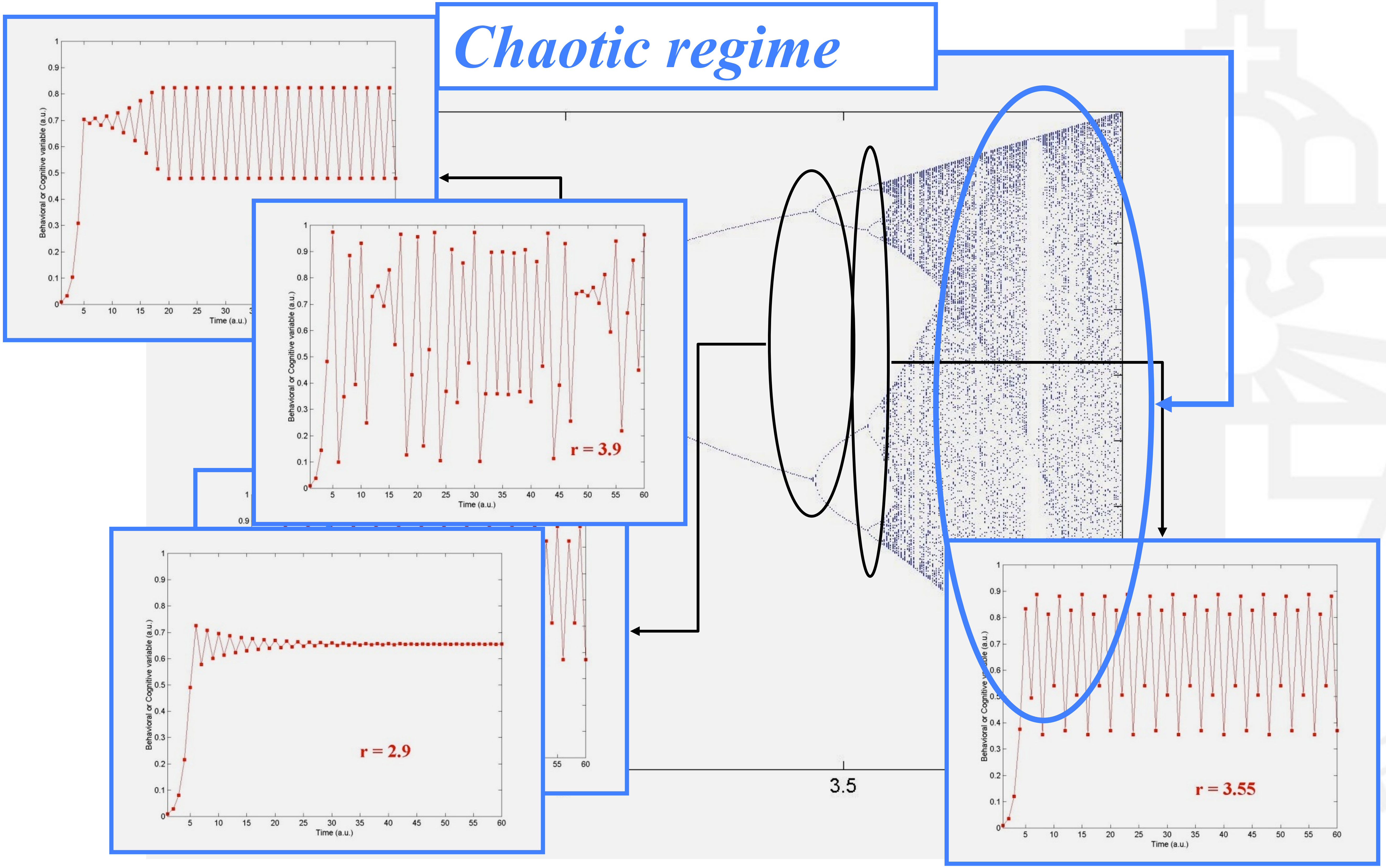
http://www.da4ga.nl/wp-content/uploads/2012/03/PastedGraphic-2-1.jpg

Logistic Map: Bifurcation Diagram

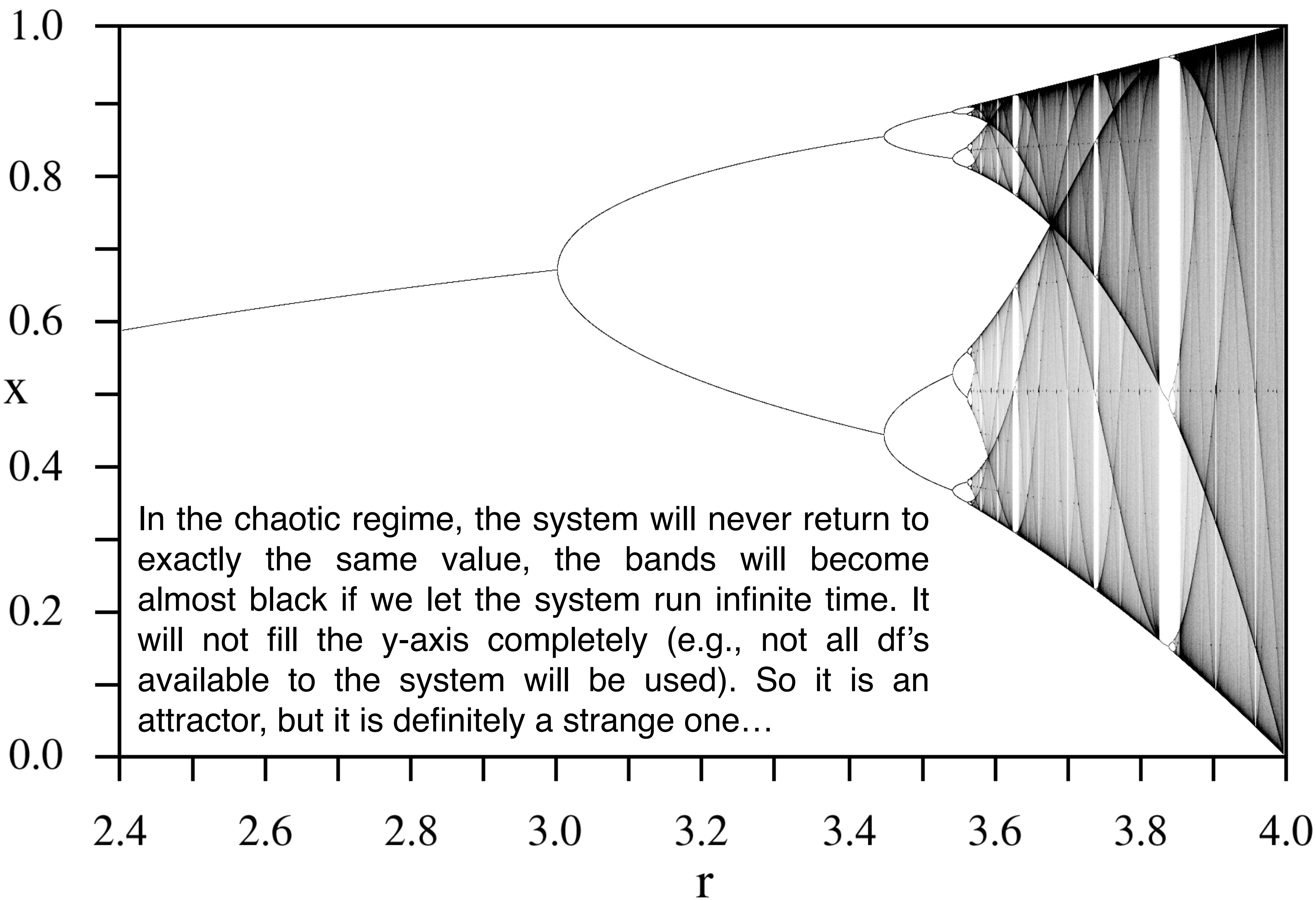


[1]refs

Chaotic regime



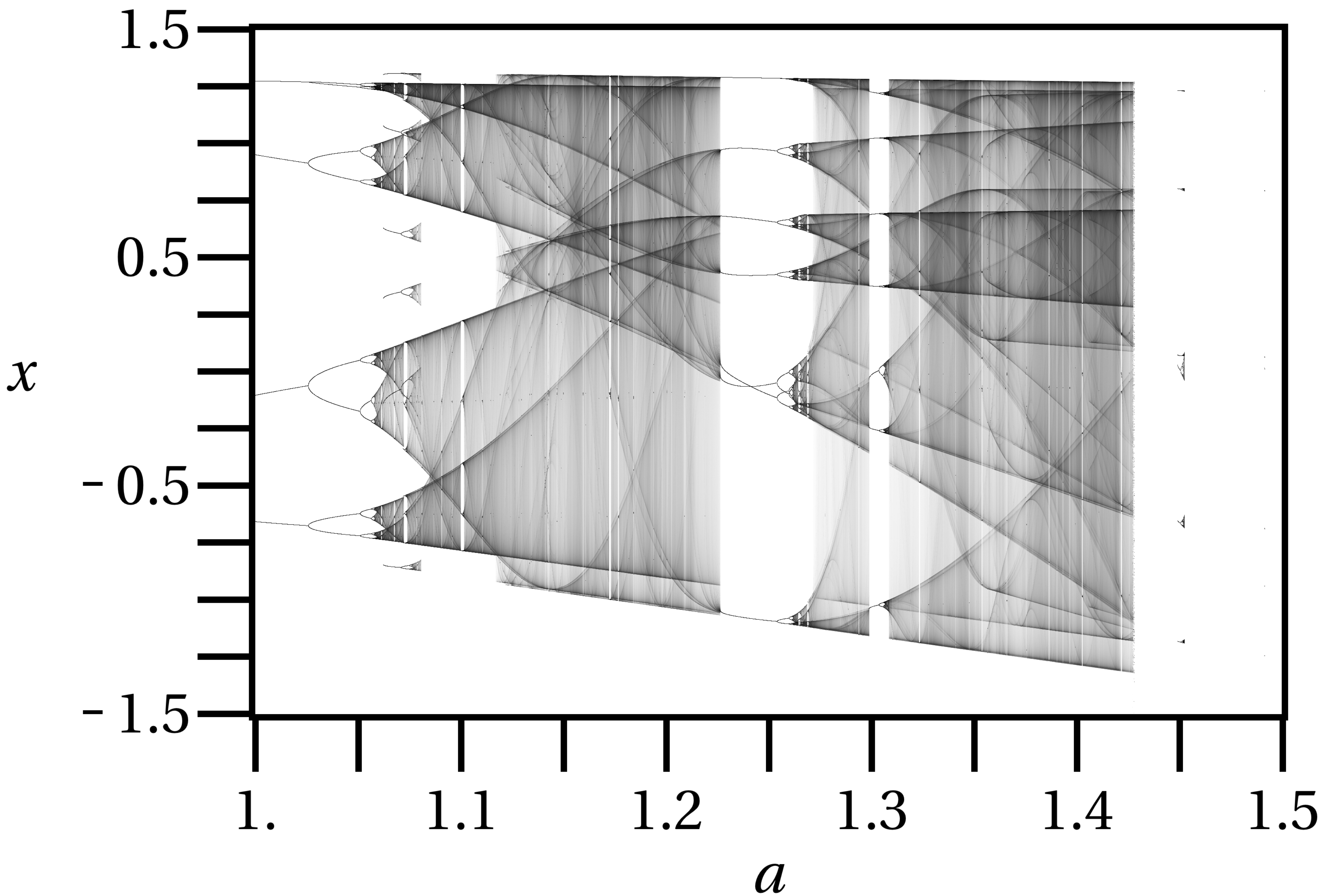
Logistic Map: Bifurcation Diagram



In the chaotic regime, the system will never return to exactly the same value, the bands will become almost black if we let the system run infinite time. It will not fill the y-axis completely (e.g., not all df's available to the system will be used). So it is an attractor, but it is definitely a strange one…

http://upload.wikimedia.org/wikipedia/commons/7/7d/LogisticMap_BifurcationDiagram.png

Henon Map: Bifurcation Diagram



http://upload.wikimedia.org/wikipedia/commons/c/cd/Henon_bifurcation_map_b%3D0.3.png

DETERMINISTIC CHAOS

CHAOS, TURBULENCE
and other unsolved mysteries

"*Turbulence is the most important unsolved problem of classical physics*"

- Richard Feynman (1918 - 1988)

"*I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment:*

One is quantum electrodynamics,
and the other is the turbulent motion of fluids.

And about the former I am rather optimistic."

- Horace Lamb (1849 - 1934)

Turbulent

Laminar

Behavioural Science Institute
Radboud University Nijmegen

Deterministic Chaos

Table 12-1. Summary of the Hierarchy of Dynamic Systems.

Type	Constraints	Description
Zero	Absolute	Constant state
I	Analytic integrals	Solvable dynamic system
II	Approximate analytic integrals	Amenable to perturbation theory
III	Quasi-deterministic; smooth but erratic trajectory	Chaotic dynamic system
IV	Rigorously defined only by averages over time or state space	Turbulent/stochastic

Table 12-2. A few examples of the types of dynamic systems.

Type	Examples
Zero	Images, gravity models, structures
I	Gear trains, 2-body problem, physical pendulum
II	Satellite orbits, lunar and planetary theories
III	Climatology, Lorenz equations, discrete logistic equation
IV	Quantum mechanics, turbulent flow, statistical mechanics

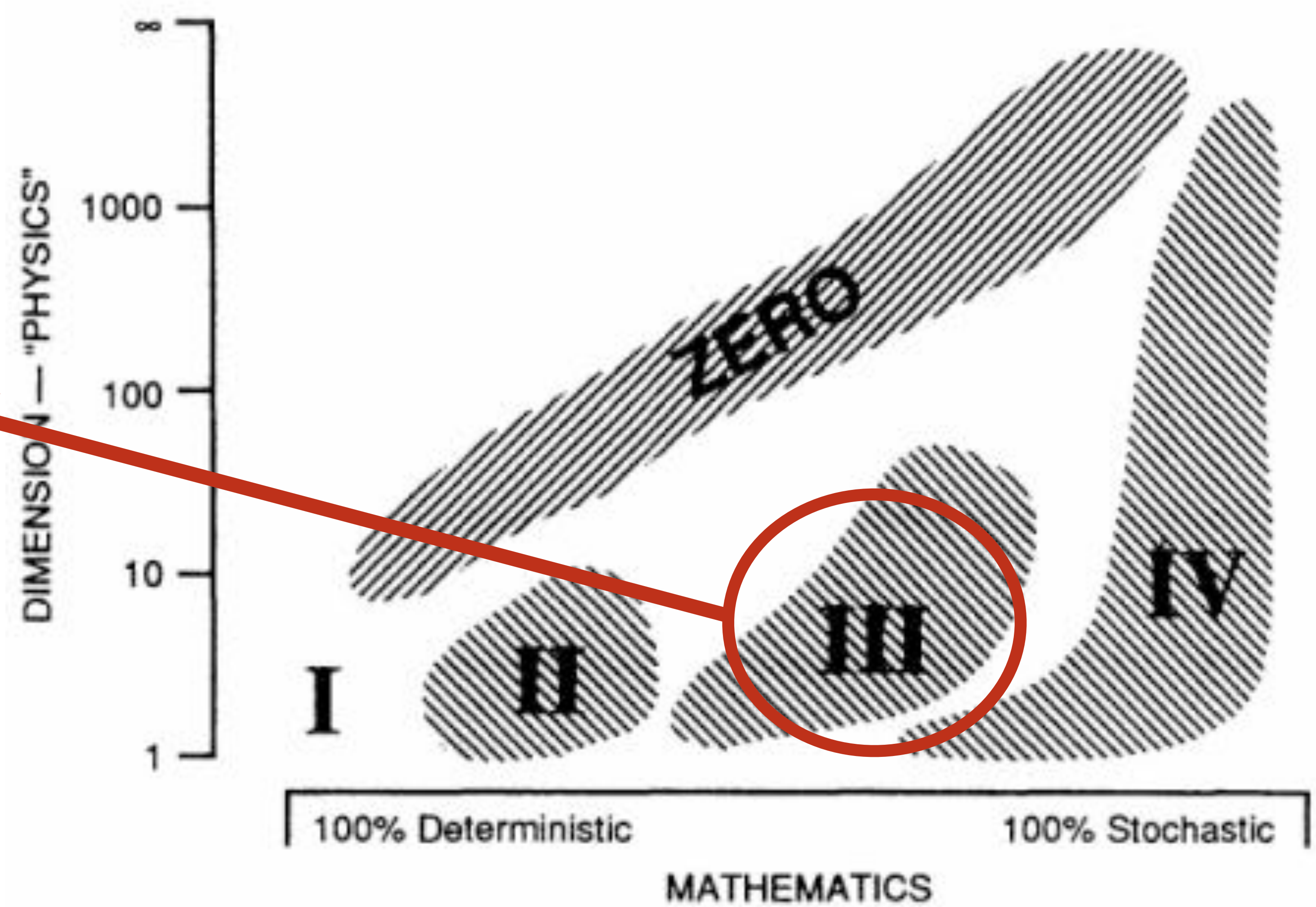


Figure 12-1. Schematic representation of the Hierarchy of Dynamic Systems.

Deterministic Chaos

There is no real definition of chaos, but there are at least four ingredients:

*The dynamics is **a-periodic** and **bounded**, and the system is **deterministic** and **sensitively depends on initial conditions**.*

[1]refs

Deterministic Chaos… Paradox?

Something that is ***deterministic***, is:

- *Mathematically exact;*
- *Predictable*.

Something that is ***’chaotic’***, shows:

- *Disorderly behaviour;*
- *Extreme sensitivity*.

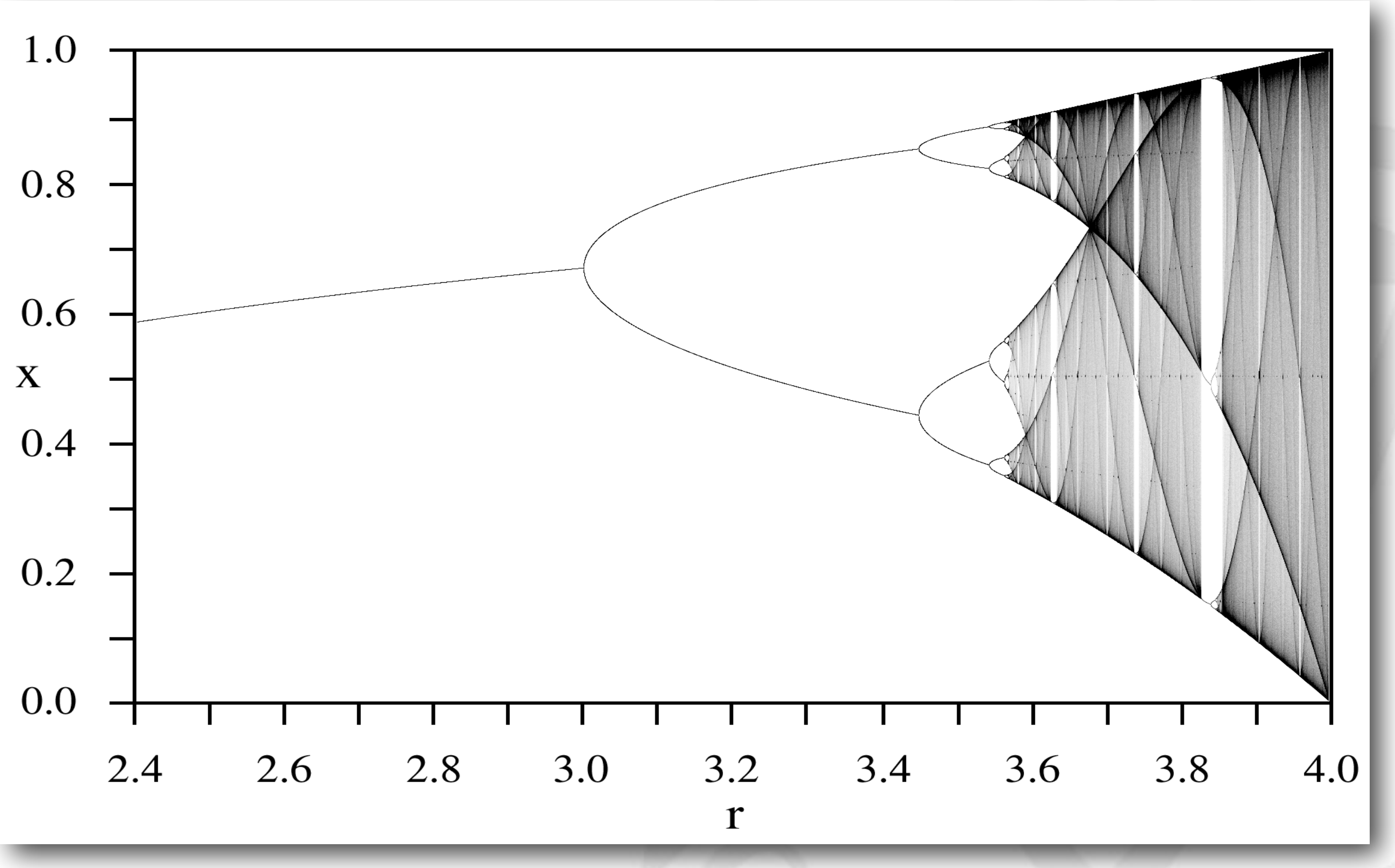
[1]refs

CHAOS, TURBULENCE
and other unsolved mysteries

Chaotic regime of the logistic map represented by the bifurcation diagram

Transitions between regimes:

- Order to Order
- Order to Chaos
- Chaos to Order
- Chaos to Chaos



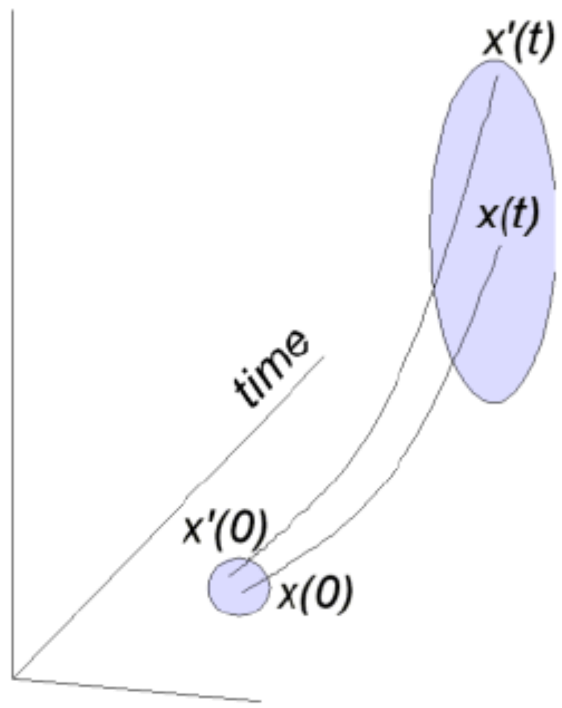
Why this happens at these parameter settings is…. unknown

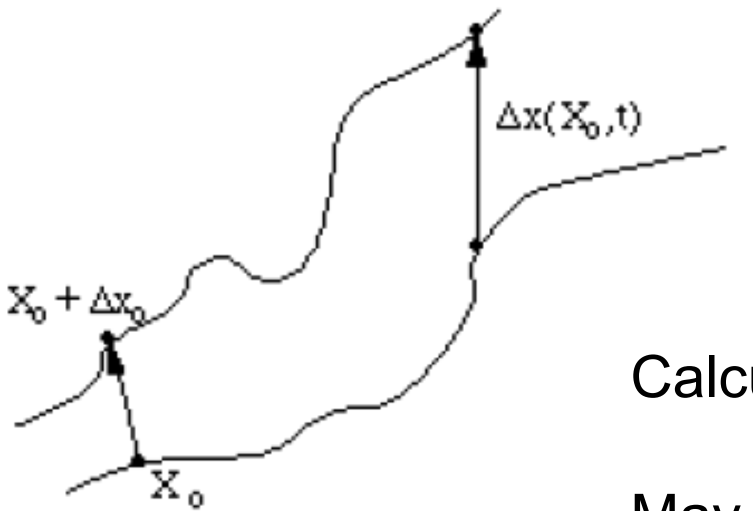
CHAOS, TURBULENCE
and other unsolved mysteries

What *can* we say about chaos?

4. Sensitive dependence on initial conditions

The *Lyapounov Exponent* characterises (quantifies) the rate of separation of two infinitesimally close trajectories in state space.





Calculate if you have a model

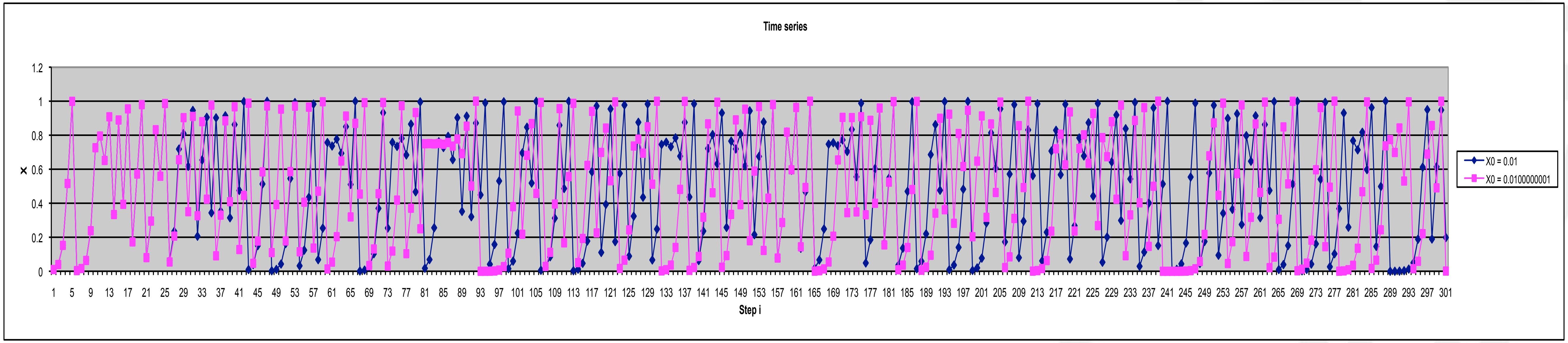
May be experimentally accessible

Analytic techniques (in R) are available

Sensitive Dependence on Initial Conditions

What *can* we say about deterministic chaos and complexity?

$X_0 = 0.01$ **$X_0 = 0.01000000001$**



Tiny differences in initial conditions can yield diverging time-evolutions of system states

Lorenz observed this in his models of the upper atmosphere:
The divergence was so extreme it resembled a butterfly flapping its wings -or not- could be the difference between weather developing as a hurricane or a summer breeze

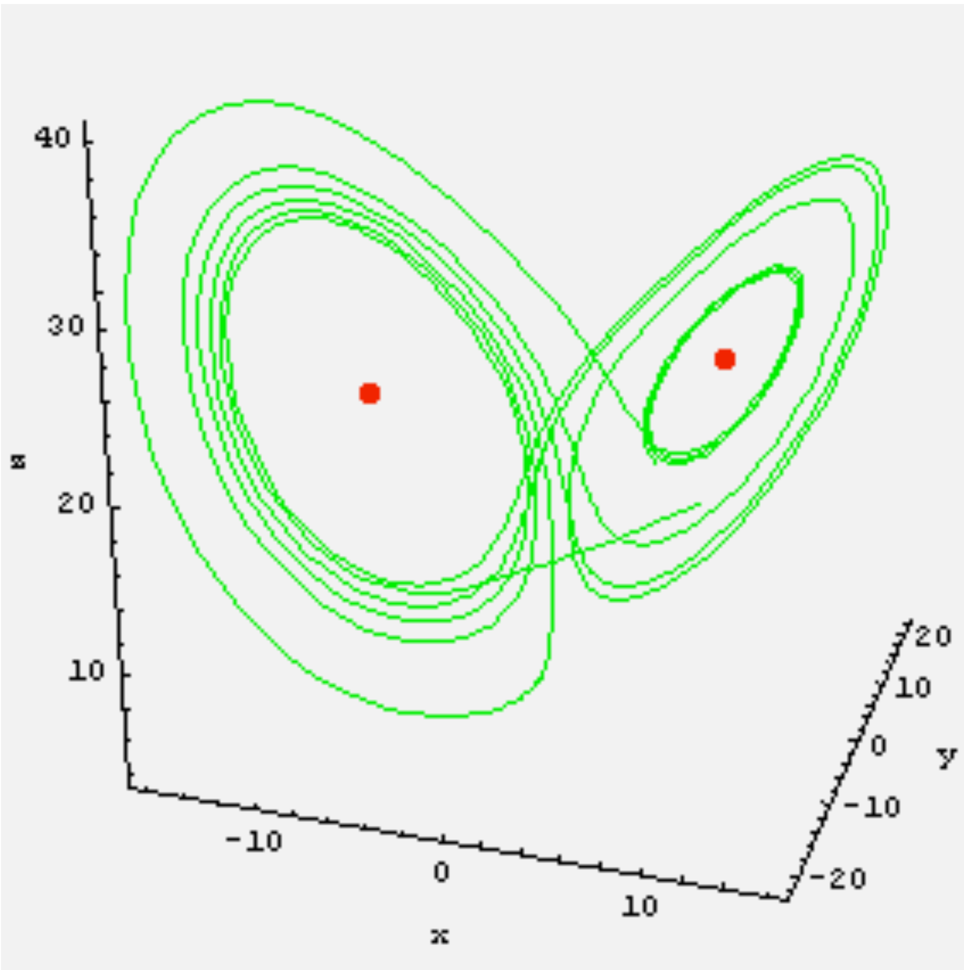
Lorenz Attractor

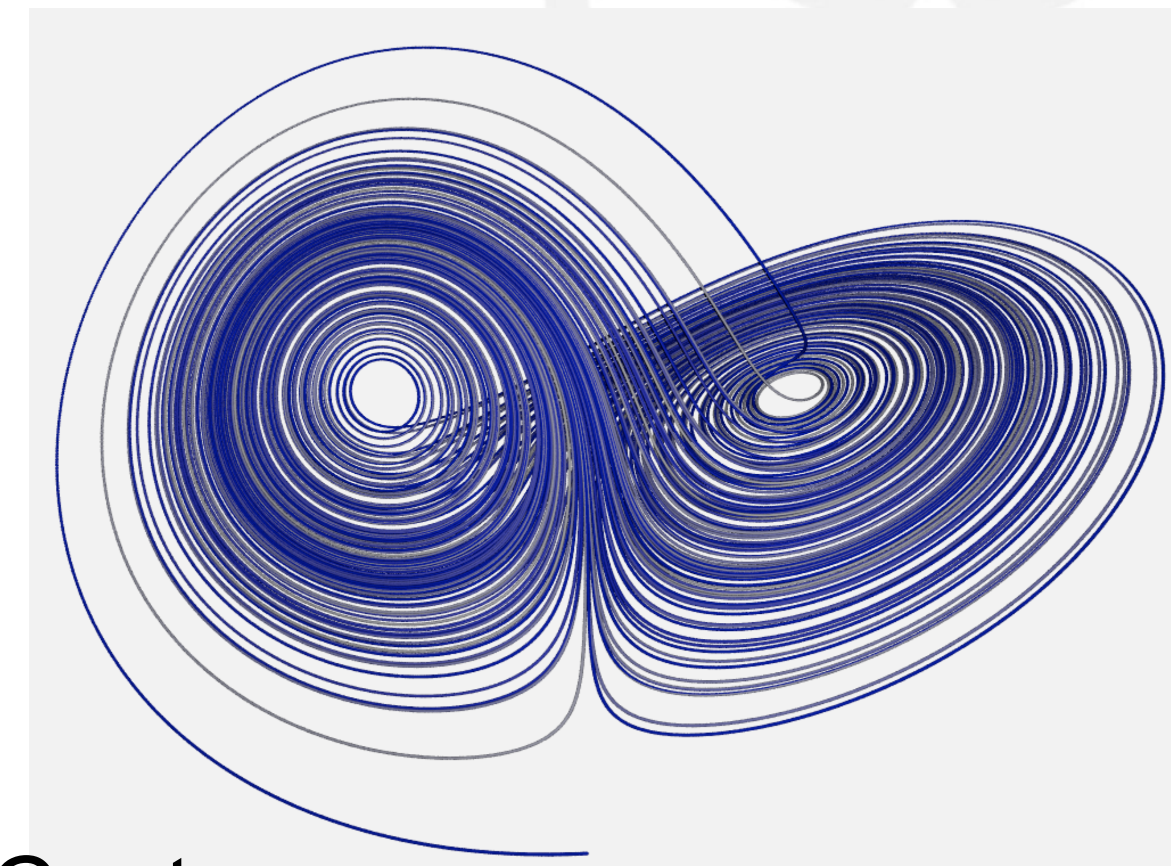
$$\frac{dx}{dt} = a(y - x),$$
$$\frac{dy}{dt} = x(b - z) - y,$$
$$\frac{dz}{dt} = xy - cz.$$

Deterministic Chaos

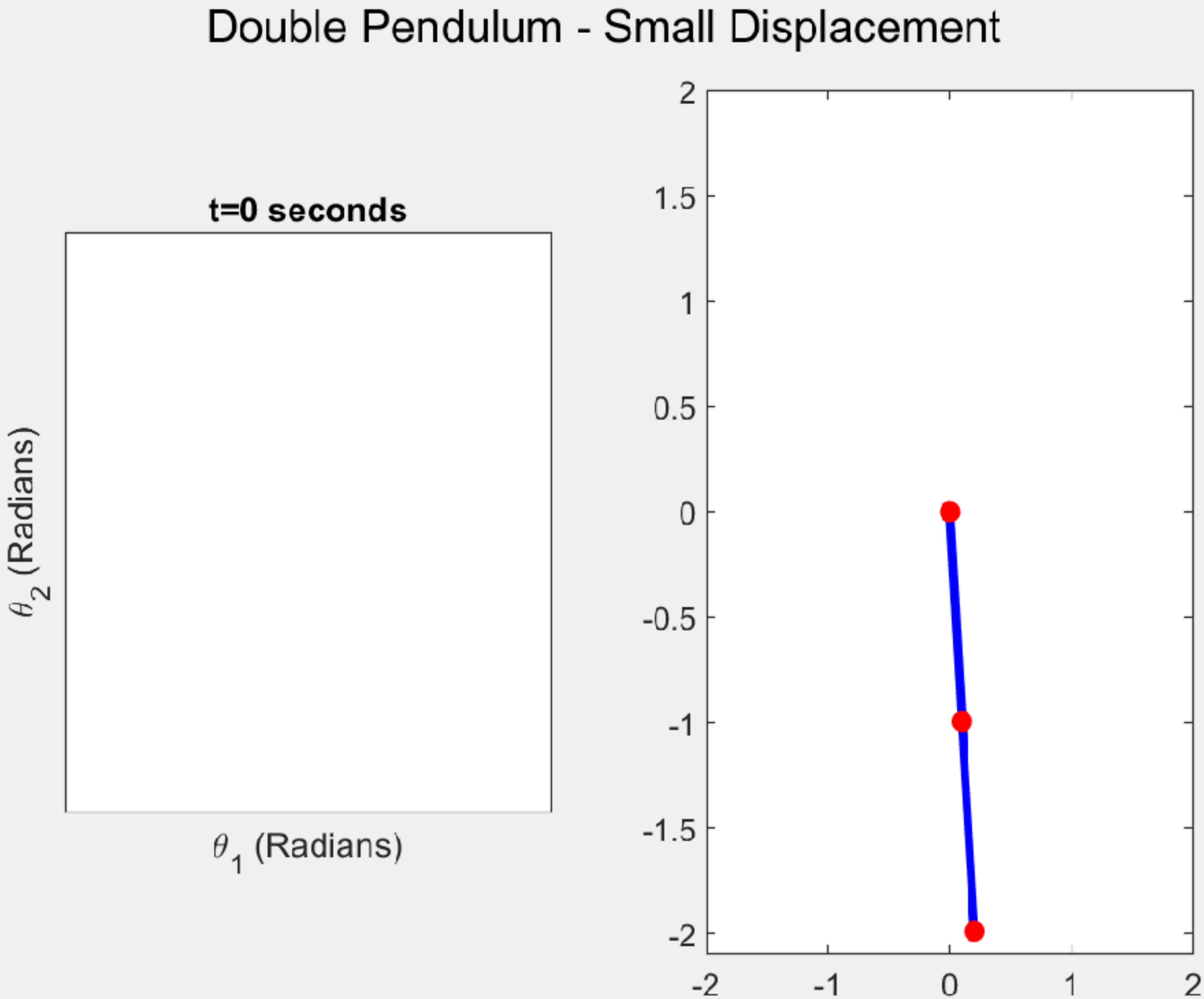
Maps: linear map, 1D state space

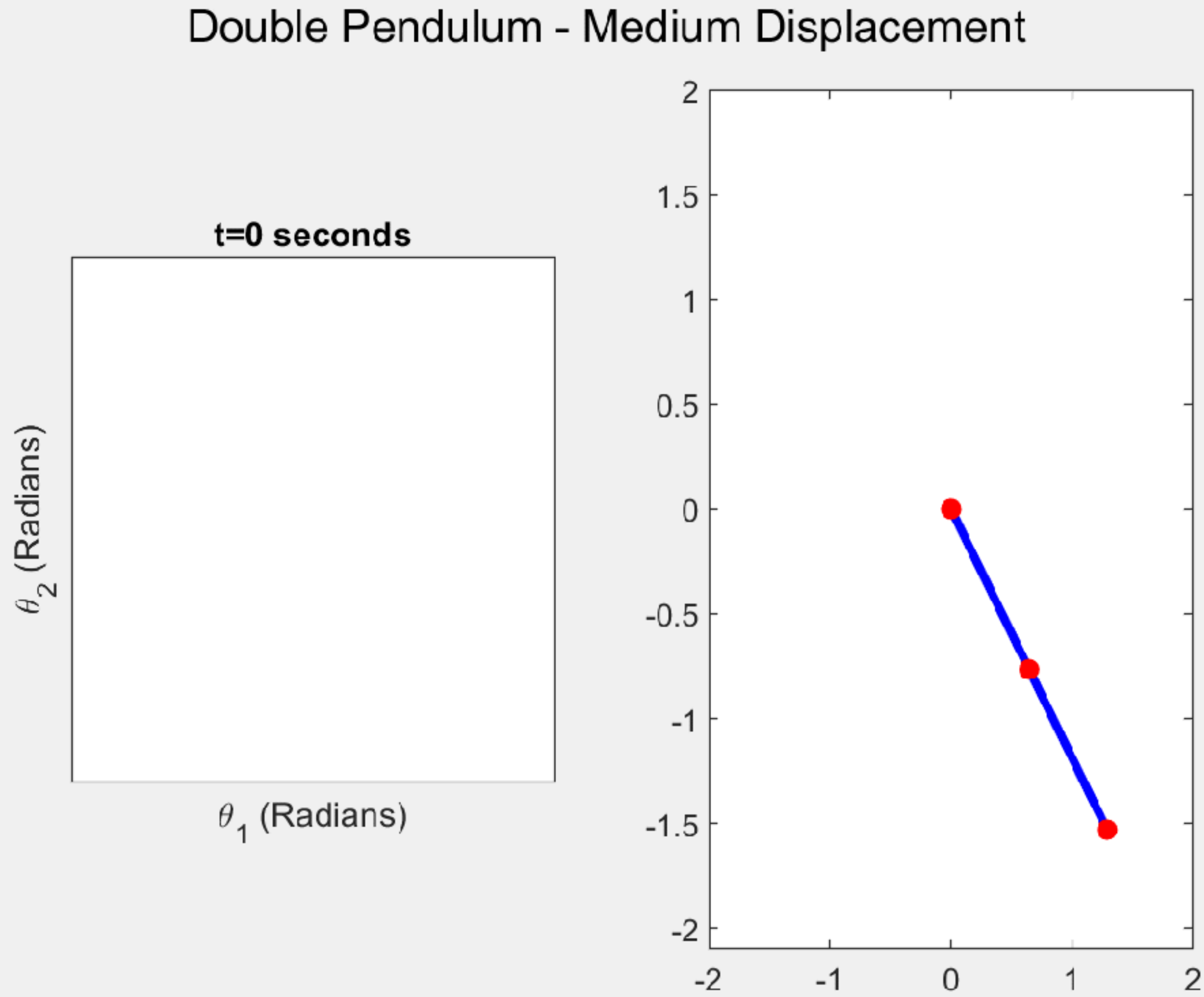
Flows: Need 3 coupled ODEs (ordinary differential equations) Minimum is 3D state space



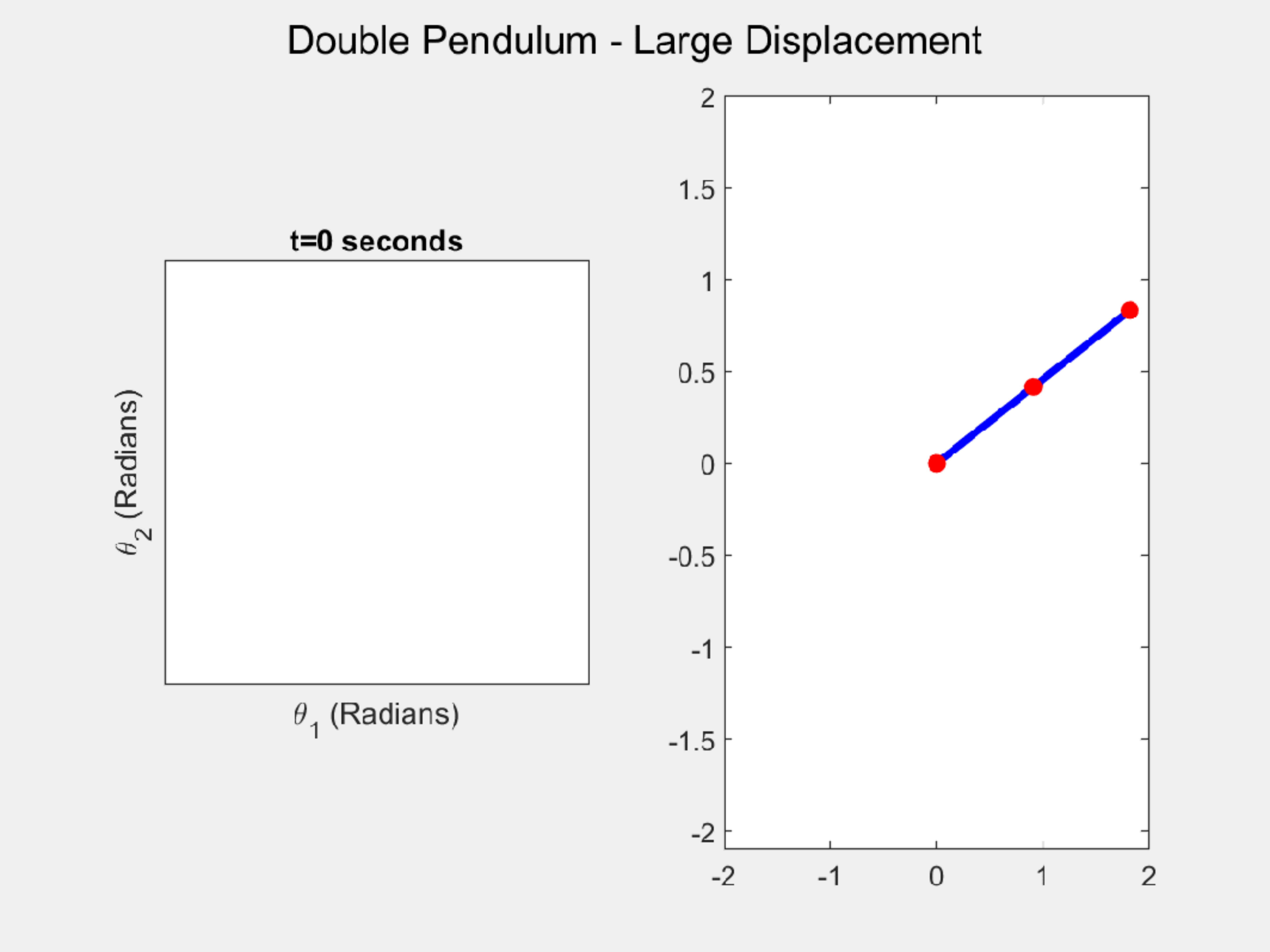


Lorenz about chaos, fractals, SOC, etc.:
"*Study of things that look random -but are not*"

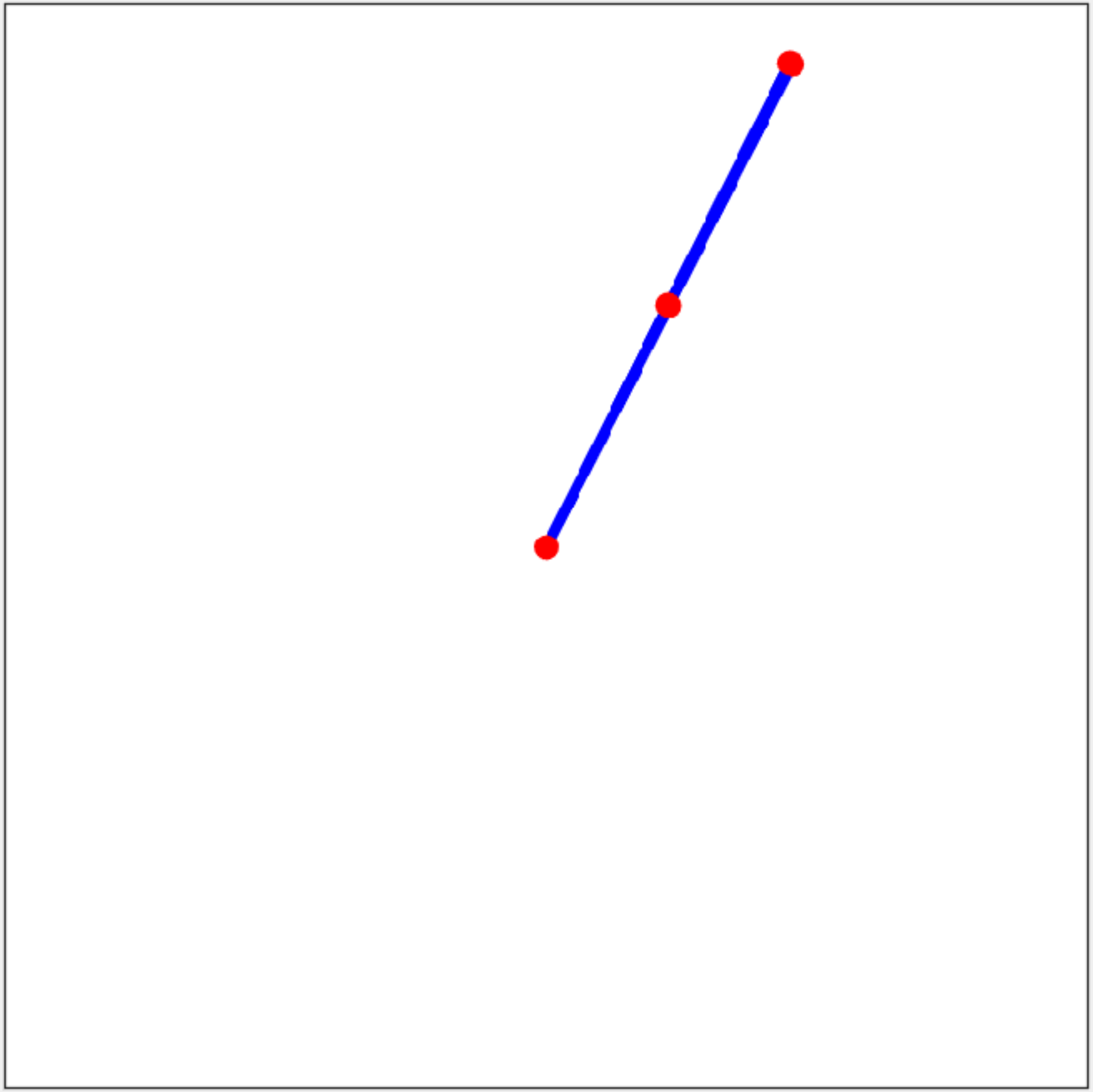

Double Pendulum - Small Displacement
t=0 seconds
θ_2 (Radians)
θ_1 (Radians)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2



Double Pendulum - Medium Displacement
t=0 seconds
θ_2 (Radians)
θ_1 (Radians)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2


Double Pendulum - Large Displacement
t=0 seconds
θ_2 (Radians)
θ_1 (Radians)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2

Double Pendulum at t=0 seconds



https://youtu.be/PrPYeu3GRLg?t=68

[1]refs